Panel 1

Anti-Derivatives

$\int f(x)\,dx$ is that function $F(x)$ such that

$$F'(x) = f(x)$$

Rules

$$\int x^p\,dx = \frac{1}{p+1}x^{p+1} + C$$

$$\int \frac{1}{x}\,dx = \ln|x| + C \qquad \int e^x\,dx = e^x + C$$

1

Panel 2

Ex: $\int 3x^2 + 4\sqrt{x} - \frac{7}{x^2} + \frac{3}{x} + 4e^x + 4\,dx$

$$= 3\tfrac{1}{3}x^3 + 4\tfrac{2}{3}x^{3/2} - 7(-1)x^{-1} + 3\ln|x| + 4e^x + 4x + C$$

$$= x^3 + \tfrac{8}{3}x^{3/2} + 7x^{-1} + 3\ln|x| + 4e^x + 4x + C$$

2

Panel 3

Name: _____

Quiz #8

Evaluate the following indefinite integrals:

a) $\int 2x^1\,dx = 2\frac{1}{2}x^2 + C = x^2 + C$

b) $\int x^4\,dx = \frac{1}{5}x^5 + C$

c) $\int 9\sqrt[3]{x} = \int 9x^{1/3}\,dx = 9\frac{3}{4}x^{4/3} + C$

d) $\int \frac{3}{2x^2} - \frac{7}{3}\sqrt{x}\,dx = \int \frac{3}{2}x^{-2} - \frac{7}{3}x^{1/2}\,dx = \frac{3}{2}(-1)x^{-1} - \frac{7}{3}\cdot\frac{2}{3}x^{3/2} + C$

3

Panel 4

(2) Find a function $y(x)$ such that $y' = 4x - \frac{3}{x}$ and $y(1) = 2$

$y = \int 4x - \frac{3}{x}\,dx = 2x^2 - 3\ln|x| + C$ Hint: $\ln(1) = 0$

$y(1) = 2 - 3\ln|1| + C = 2$

$C = 0$

$y(x) = 2x^2 - 3\ln(x)$

(3) If a marginal cost function is $C'(q) = 10q - 3q^2$ and the fixed cost is \$10, find the total cost for producing $q = 3$ items.

$C(q) = \int 10q - 3q^2\,dq = 10\frac{1}{2}q^2 - 3\frac{1}{3}q^3 + C = 5q^2 - q^3 + C$

Know: $C(0) = C = 10$ $C(q) = 5q^2 - q^3 + 10$, $C(3) = 45 - 27 + 10 = 28$

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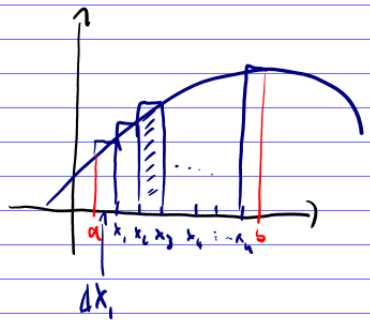
Panel 5

Definite Integral: $\int_a^b f(x)\,dx = \lim_{n\to\infty}\left[f(x_1)\Delta x_1 + f(x_2)\Delta x_2 + \dots f(x_n)\Delta x_n\right]$

$\int_a^b$ ← integral of f from a to b



Remember that $\int_a^b f(x)dx$ is defined via a lim of a sum of rectangles

Fundamental Theorem of Calculus: If f is continuous

then $\int_a^b f(x)\,dx = F(b)-F(a)$ where F is an antiderivative

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Panel 6

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b)-F(a)$$

Ex: $\int_0^2 2x\,dx = x^2\Big|_0^2 = (2)^2 - 0^2 = 4$

$$\int_0^1 5e^x - 7\sqrt[3]{x^5}\,dx = 5e^x - 7\cdot\tfrac{3}{8}x^{8/3}\Big|_0^1 =$$

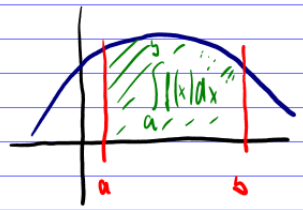
$-7\ x^{5/3}$

$\left(5e - \frac{21}{8}\right) - \left(5e^0 - 0\right) = 5e - \frac{21}{8} - 5$

6

Panel 7

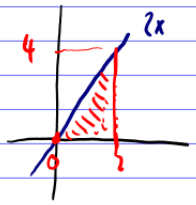
Interpretation of Definite Integral



$\int_a^b f(x)\,dx$ is area under curve $f(x)$ from a to b, as long as $f \geq 0$

Ex: $\int_0^2 2x\,dx$ graphically and algebraically

C is not necessary



$\int_0^2 2x\,dx = \frac{1}{2}\cdot 2\cdot 4 = 4$ or $\int_0^2 2x\,dx = x^2 + C\Big|_0^2 = (4+C)-(0+C)$

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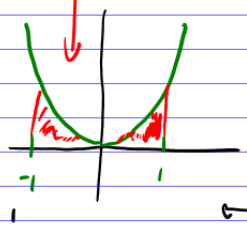
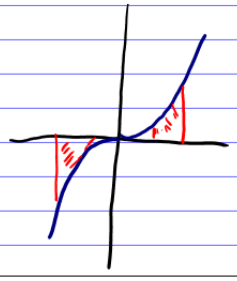
Panel 8

Ex: $\int_{-1}^{1} x^2\,dx = \frac{1}{3}x^3\Big|_{-1}^{1} = \frac{1}{3}(1)^3 - \frac{1}{3}(-1)^3 =$

$= 1/3 + 1/3 = 2/3$



Ex: $\int_{-1}^{1} x^3\,dx = \frac{1}{4}x^4\Big|_{-1}^{1} = \frac{1}{4}(1)^4 - \frac{1}{4}(-1)^4 = 0$ strange as even



because f is positive and negative, so $\int_a^b f(x)\,dx$ is "signed" area, not true area.

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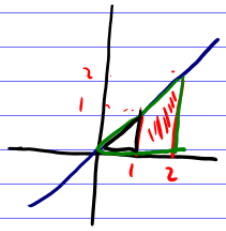
Panel 9

$\int_1^2 x\,dx$ graphically



Area = green triangle − black triangle

$\frac{1}{2}\cdot 2\cdot 2 + \frac{1}{2}\cdot 1\cdot 1 = 2 + \frac{1}{2} = \frac{3}{2}$

$$\int_1^2 x\,dx = \frac{1}{2}x^2\Big|_1^2 = \frac{1}{2}(2)^2 - \frac{1}{2}(1)^2 = 2 - \frac{1}{2} = \frac{3}{2}$$

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