Panel 1

Had fun with Derivatives ....

1

Panel 2

Anti-Derivatives

Done: Take $f(x)$, find $f'(x)$ (using power rule)

Opposite: Start with $f(x)$, find $F(x)$ such that $F'(x) = f(x)$

Ex: $f(x) = 2x \Rightarrow F(x) = x^2$, because $\frac{d}{dx}(x^2) = 2x$

That F is called Anti-derivative of f.

Notation: $\int f(x)\,dx = F(x)$, i.e. $\int 2x\,dx = x^2$

integral of $f(x)\,dx$

2

Panel 3

Ex:

$$\int 1\,dx = x + C, \quad \frac{d}{dx}(x) = 1$$

$$\int x\,dx = \frac{1}{2}x^2 + C, \quad \frac{d}{dx}\left(\frac{1}{2}x^2\right) = \frac{1}{\cancel{2}}\cancel{2}x = x$$

$$\int x^5\,dx = \frac{1}{6}x^6 + C,$$

$$\int x^8\,dx = \frac{1}{9}x^9 + C$$

$$\int \sqrt{x}\,dx = \int x^{1/2}dx = \frac{2}{3}x^{3/2} + C, \quad \frac{d}{dx}\left(\frac{2}{3}x^{3/2}\right) = \frac{\cancel{2}}{\cancel{3}}\cdot\frac{\cancel{3}}{\cancel{2}}\cdot x^{1/2} \checkmark$$

$$\int \frac{1}{x^3}\,dx = \int x^{-3}dx = -\frac{1}{2}x^{-2} + C \checkmark$$

$$\int \frac{1}{\sqrt[3]{x^4}}\,dx = \int x^{-4/3}dx = -3\,x^{-1/3} + C$$

3

Panel 4

Anti Power Rule

$$\int x^p\,dx = \frac{1}{p+1}\,x^{p+1} + C$$

Ex: $$\int x^7\,dx = \frac{1}{8}x^8 + C$$

$$\frac{d}{dx}\left(\frac{1}{8}x^8 + 2\right) = \frac{1}{8}8x^7 = x^7$$

Note: $\int f(x)\,dx$ usually involves a constant!

4

Panel 5

More:

$$\int x^2 + 2x \, dx$$

$$\frac{1}{3}x^3 + 2\frac{1}{2}x^2 = \frac{1}{3}x^3 + x^2 + C$$

$$\int 2\sqrt[5]{x^4} - 7x^3 + 10 \, dx$$

$$\int 2 x^{4/5} - 7x^3 + 10x^0 \, dx =$$

$$= 2\frac{5}{9}x^{9/5} - 7\frac{1}{4}x^4 + 10x + C$$

$$2\frac{1}{\frac{9}{5}}x^{9/5}$$

5

Panel 6

Even more:

① Find a function y such that $y' = 8x - 4$ and $y(2) = 5$

initial condition

Want: $y = \int 8x - 4 \, dx = 8\frac{1}{2}x^2 - 4x + C =$

$$= 4x^2 - 4x + C$$

$x = 2$

Want: $y(2) = 5$

$$y(2) = 4(2)^2 - 4(2) + C = 8 + C = 5 \Rightarrow C = -3$$

$$\boxed{y(x) = 4x^2 - 4x - 3}$$

6

Panel 7

(i) Find a function y such that $y'' = x^2 - 6$ and also $y'(0) = 2$ and $y(1) = -1$

$$y' = \int x^2 - 6 \, dx = \frac{1}{3}x^3 - 6x + C$$

$$y'(0) = 0 + 0 + C = 2$$

$$\Rightarrow \underline{y'(x) = \frac{1}{3}x^3 - 6x + 2}$$

$$y = \int \frac{1}{3}x^3 - 6x + 2 \, dx = \frac{1}{3}\frac{1}{4}x^4 - 6\frac{1}{2}x^2 + 2x + D$$

$$y(x) = \frac{1}{12}x^4 - 3x^2 + 2x + D$$

$$y(1) = \frac{1}{12} - 3 + 2 + D = -1$$

$$-\frac{11}{12} + D = -1 \Rightarrow D = -\frac{1}{12}$$

7

Panel 8

Ex: Suppose the marginal revenue function for a product is $\frac{dr}{dq} = 2000 - 20q - 3q^2$

Find the demand function

$$R(q) = \int 2000 - 20q - 3q^2 \, dq = 2000q - 20\frac{1}{2}q^2 - 3\frac{1}{3}q^3 + C$$

$$= 2000q - 10q^2 - q^3 + C$$

$$R(0) = \underline{0} = C \Rightarrow R(q) = 2000q - 10q^2 - q^3 = \underline{q(2000 - 10q - q^2)}$$

$$R(q) = p(q) \cdot q = q(2000 - 10q - q^2) \Rightarrow p(q) = \underline{2000 - 10q - q^2}$$

demand/supply

8

Panel 9

Rules of Integration

$$\int x^p \, dx = \frac{1}{p+1} x^{p+1} + C, \quad p \neq -1$$

new $\int \frac{1}{x} \, dx = \ln|x| + C$

easy $\int e^x \, dx = e^x + C$

sums / differences are done separately

constants · functions = keep const., find antideriv. of function

9

Panel 10

$$\int 7x^3 + \overset{3x^{-2}}{\frac{3}{x^2}} + \frac{2}{x} + 7e^x + \pi \, dx$$

$$7 \cdot \frac{1}{4} x^4 + 3\left(\frac{1}{-1}\right) x^{-1} + 2 \ln|x| + 7e^x + \pi x + C$$

10

Panel 11

e) $\int \frac{2x^2}{7} - \frac{8}{3} x^4 \, dx = \frac{2}{7}\frac{1}{3}x^3 - \frac{8}{3}\frac{1}{5}x^5 + C$ ✓

f) $\int (x^2+5)(x-3)\,dx = \int x^3 - 3x^2 + 5x - 15\,dx = \frac{1}{4}x^4 - 3\frac{1}{3}x^3 + 5\frac{1}{2}x^2 - 15x + C$

g) $\int \frac{2}{\sqrt{x^3}} - \frac{\sqrt[4]{x^5}}{8}\,dx \quad \int 2x^{-3/2} - \frac{1}{8}x^{5/4}\,dx = 2(-2)x^{-1/2} - \frac{1}{8}\frac{4}{9}x^{9/4} + C$

h) $\int (3x+2)^3\,dx$ Fool

11

Panel 12

a) $\int \frac{7}{x}\,dx = 7 \cdot \ln|x| + C$

b) $\int 9e^x\,dx = 9e^x + C$

c) $\int 5e^x + \frac{1}{3x}\,dx = 5e^x + \frac{1}{3}\ln|x| + C$

$\frac{1}{3x} = \frac{1}{3}\frac{1}{x}$

12

Panel 13

If the fixed costs for producing Acme Quality Widgets is $2000 and the marginal cost is $c'(q) = 0.08q^2 - 1.6q + 6.5$, find the cost for producing 25 units.

$$C(q) = \int 0.08q^2 - 1.6q + 6.5 \, dq = 0.08 \tfrac{1}{3} q^3 - 1.6 \tfrac{1}{2} q^2 + 6.5q + D$$

$$C(q) = \frac{0.08}{3} q^3 - 0.8q^2 + 6.5q + D$$

$$C(0) = 0 + D = 2000 \Rightarrow D = 2000$$

$$C(q) = \frac{0.08}{3} q^3 - 0.8q^2 + 6.5q + 2000 \Rightarrow C(25) = \text{calculator}$$

13

Panel 14

Are done with $\int f(x)\,dx$, also called the indefinite integral

Next: Definite Integral $\int_a^b f(x)\,dx$ integral of $f(x)\,dx$ as $x=a$ to $x=b$

Ex: $\int_0^1 x^2 \, dx =$ Next time.

Quiz on Wed!

14