Panel 1

Review

$\lim_{x\to 3} \frac{x^2-9}{x-3}$

Limits: try to substitute, or if 0/0 try to factor
one sided limits, limits at infinity, graphically

Continuity: (1) $f(c)$ exists
(2) $\lim_{x\to c} f(x)$ exists
(3) are same

graph no hole or gap.

Derivative: $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$, power rule, higher-order derivs.

f': slope of tangent, inst. rate of change, marginal X, velocity,
incr. + decreasing of f ⇒ max and mins

f'': tells about concavity up/down, inflection points

1

Panel 2

$$f(x) = \begin{cases} 2x-5 & \text{if } x<2 \\ -x & \text{if } x \geq 2 \end{cases}$$

cont. at $x=0$
(1) $f(0) = -5$
(2) $\lim_{x\to 0} f(x) = -5$
(3) yes

cont. at $x=2$: No!
(1) $f(2) = -2$
(2) $\lim_{x\to 2} f(x) =$ undef
(3)

$\lim_{x\to 2^+} (-x) = -2$

$\lim_{x\to 2^-} (2x-5) = -1$

2

Panel 3

$$f(x) = \begin{cases} x^2-1 & x \le 2 \\ kx & x > 2 \end{cases}$$

① $f(2) = 3$ Yes if $k = 3/2$

② $\lim_{x \to 2^+} (kx) = 2k$ Need $2k = 3 \Rightarrow \boxed{k = 3/2}$ limit exists and is 3

$\lim_{x \to 2^-} (x^2-1) = 3$

$\lim_{x \to 2} f(x) = 3$

③ $3 = 3$

3

Panel 4

$f(x) = x^2 - 6x + 3$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{[(x+h)^2 - 6(x+h) + 3] - [x^2 - 6x + 3]}{h} =$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{6x} - 6h + \cancel{3} - \cancel{x^2} + \cancel{6x} - \cancel{3}}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 - 6h}{h} = \lim_{h \to 0} \frac{\cancel{h}(2x + h - 6)}{\cancel{h}}$$

$$= \lim_{h \to 0} 2x + h - 6 = 2x - 6$$

4

Panel 5

Average between $t=1$ and $t=3$ for $f(t)$

$$\frac{f(3)-f(1)}{3-1} = \frac{\text{rise}}{\text{run}} = \frac{\text{difference in } y}{\text{difference in } x}$$

$f(x) = x^2 - 6x + 3$ equation of tangent line at $\underline{x=2}$?

Know: $f'(x) = 2x-6$. Thus: $f'(2) = -2$, $y = f(2) = -5$

$$y + \underbrace{5} = \underbrace{-2}(x - \underbrace{2})$$

5

Panel 6

$$f(t) = 4\ln(t) + \sqrt[5]{t^2} + 4e^{\pi}$$

$$t^{2/5}$$

$$f'(x) = 4\cdot\frac{1}{t} + \frac{2}{5}t^{-3/5}$$

6

Panel 7

$f(x) = x^3 + x^2 - 5x - 5$

concavity:

$f'(x) = 3x^2 + 2x - 5$

$f''(x) = 6x + 2 = 0$, $x = -1/3$

	-1	$-1/3$	0
f''	$-$		$+$
f	$\cap$		$\cup$

The function is concave up on $(-1/3, \infty)$

concave down on $(-\infty, -1/3)$

an inflection point is $x = -1/3$

Panel 8

$f(x) = x^3 - 9x^2 + 15x - 4$ incr/decr/max/min

$f'(x) = 3x^2 - 18x + 15$

$= 3(x^2 - 6x + 5) = 3(x-5)(x-1) = 0$, $x = 1, 5$

	0	1	2	5	6
f'	$+$		$-$		$+$
f	$\nearrow$		$\searrow$		$\nearrow$

f is increasing on $(-\infty, 1) \cup (5, \infty)$

decreasing on $(1, 5)$

has max at $x = 1$

min at $x = 5$