

Panel 1

Last time: Max / Min

- ① $f'(x)$
- ② solve $f'(x) = 0$ or $f'(x)$ does not exist
critical points
- ③

		(CP)	(CP)	
f'	+	-	+	
f	↗	↘	↗	
		max	min	

1

Panel 2

First vs Second Derivative

f'
 / incr. / decreasing
 \ max / min
 \ rate of change

f'' is rate of change of f'

Say $s(t)$ is a distance function depending on time t

$\Rightarrow s'(t) = v(t)$ velocity

$s''(t) = v'(t) = \underline{a(t)}$ acceleration ↙ second deriv.

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Panel 3

Ex: Suppose $s(t) = -16t^2 + 10t + 5$ is the distance function for some object. Find initial height: $s(0) = 5$

a) velocity and acceleration after 10 seconds

$$v(t) = s'(t) = -32t + 10 \Rightarrow v(10) = -310$$

$$a(t) = s''(t) = -32 \quad (\text{gravity})$$

b) When is the velocity zero? Interpret!

$$v(t) = -32t + 10 = 0 \Rightarrow t = \frac{10}{32}$$

↑
critical point.

at that time it reaches top

3

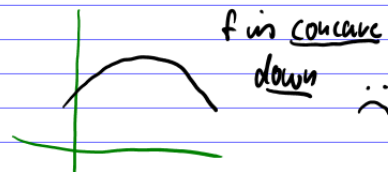
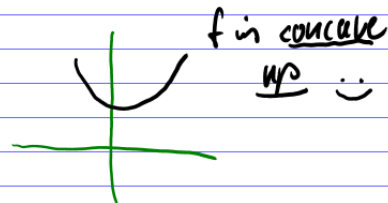
Panel 4

Meaning of f''

$f'' > 0 \Rightarrow f'$ (goes up) increasing

$f'' < 0 \Rightarrow f'$ (goes down) decreasing

$f'' = 0 \Rightarrow$ possible inflection points

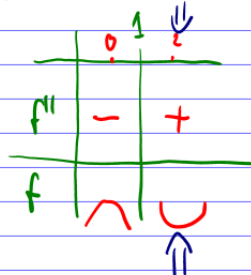


Ex: $f(x) = 2x^3 - 6x^2$ investigate concavity. Where is it concave up? $(1, \infty)$

$$f'(x) = 6x^2 - 12x$$

$$f''(x) = 12x - 12 = 0$$

$x = 1$ possible inf. point.



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Panel 5

Questions about increasing/decreasing/max/min

- ① $f'(x)$
- ② $f'(x) = 0$
- ③ table with signs of f'

Questions about concavity/inflection points

- ① $f''(x)$
- ② $f''(x) = 0$
- ③ table with signs of f''

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Panel 6

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$$C(x) = 25000 \text{ fixed cost}$$

Show that avg. cost is decreasing.

$$c(q) = \frac{C(q)}{q} \text{ is avg. cost function}$$

$$c(q) = \frac{25000}{q} = 25000q^{-1}$$

$$c'(q) = -\frac{25000}{q^2} < 0 \text{ negative always}$$

$\Rightarrow c(q)$ is always decreasing.

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Panel 7

Ex: $f(x) = x^2 - 50 \cdot \ln(x)$, $x > 1$

① For what interval(s) is f increasing?

$$f'(x) = 2x - \frac{50}{x} = 0$$

$$2x = \frac{50}{x}$$

$$x^2 = 25$$

$$x = \pm 5$$

② Where is f concave up? *for $x > 1$*

$$f''(x) = 2 + \frac{50}{x^2} = 0$$

$$2 = -\frac{50}{x^2}$$

$$x^2 = -25 \Rightarrow \text{no solution}$$

f'	-	+
f''	↘	↗

increasing on $(5, \infty)$

f''	+
f	∪

$$2x - \frac{50}{x} = 2x - 50x^{-1}$$

$$\Rightarrow \frac{d}{dx}(2x - 50x^{-1}) = 2 + 50x^{-2} = 2 + \frac{50}{x^2}$$

Panel 8

Recipe for Curve Sketching: $f(x) = x^3 - 9x^2 + 15x - 4$

① Find f'
Solve $f' = 0$

② Find f''
Solve $f'' = 0$

③

f'				
f''				
f				

④ Find $f(x)$ at special points.

① $f'(x) = 3x^2 - 18x + 15 = 3(x^2 - 6x + 5) = 3(x-5)(x-1) = 0 \Rightarrow x = 1, 5$ *critical*

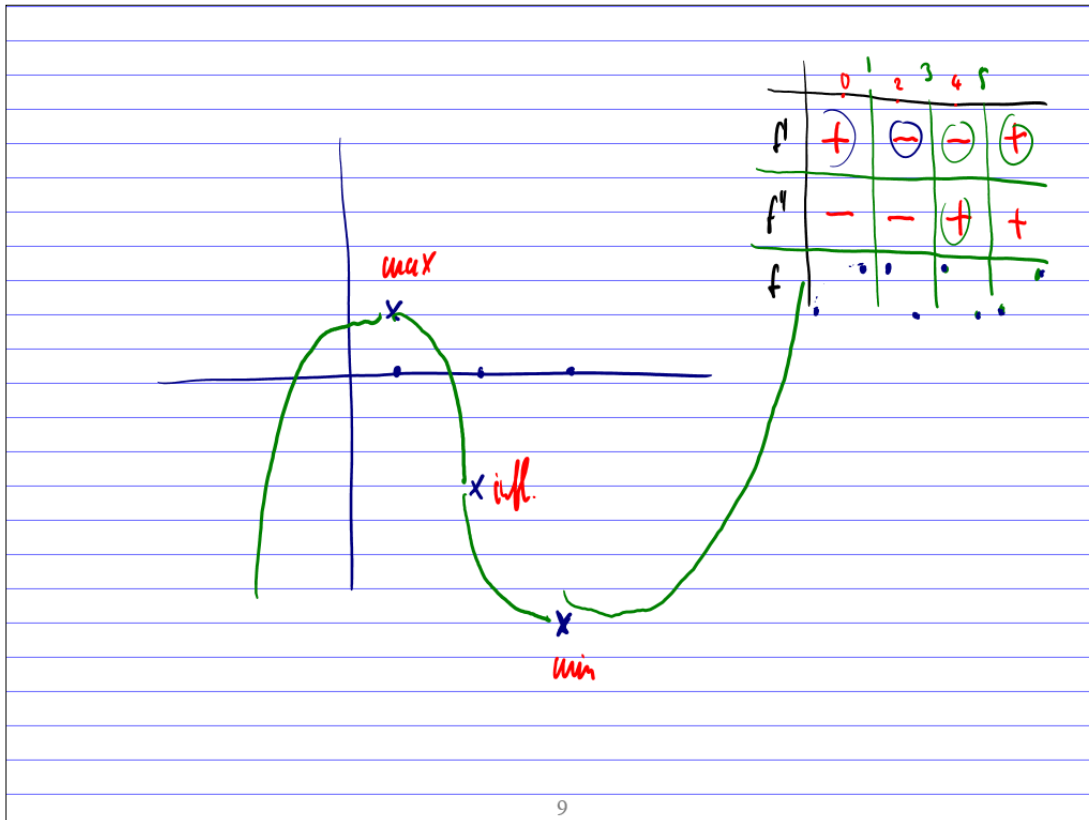
② $f''(x) = 6x - 18 = 0 \Rightarrow x = 3$ *poss. inf. point.*

③

	0	1	2	3	4	5
f'	+	+	-	-	+	+
f''	-	-	-	+	+	+
f	∪	∪	∩	∩	∪	∪

④ $f(1) = 3$
 $f(3) = -13$
 $f(5) = -29$
 $f(0) = -4$

Panel 9



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Panel 10

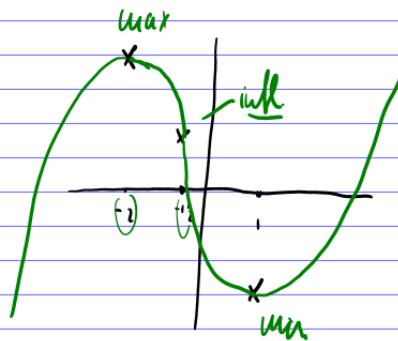
Carefully sketch the graph of $y = 2x^3 + 3x^2 - 12x - 3$. Identify all critical points and points of inflection. State the intervals over which the graph is increasing, decreasing. Concave up and concave down. Identify any absolute or relative extrema. (8%).

① $f'(x) = 6x^2 + 6x - 12 = 0$
 $= 6(x^2 + x - 2) = 0$
 $= 6(x+2)(x-1) = 0$
 $x = -2, 1$

	-3	-2	-1	0	1	max/min
f'	+	-	-	+		
f''	-	-	+	+		
f		↘	↘	↘	↘	

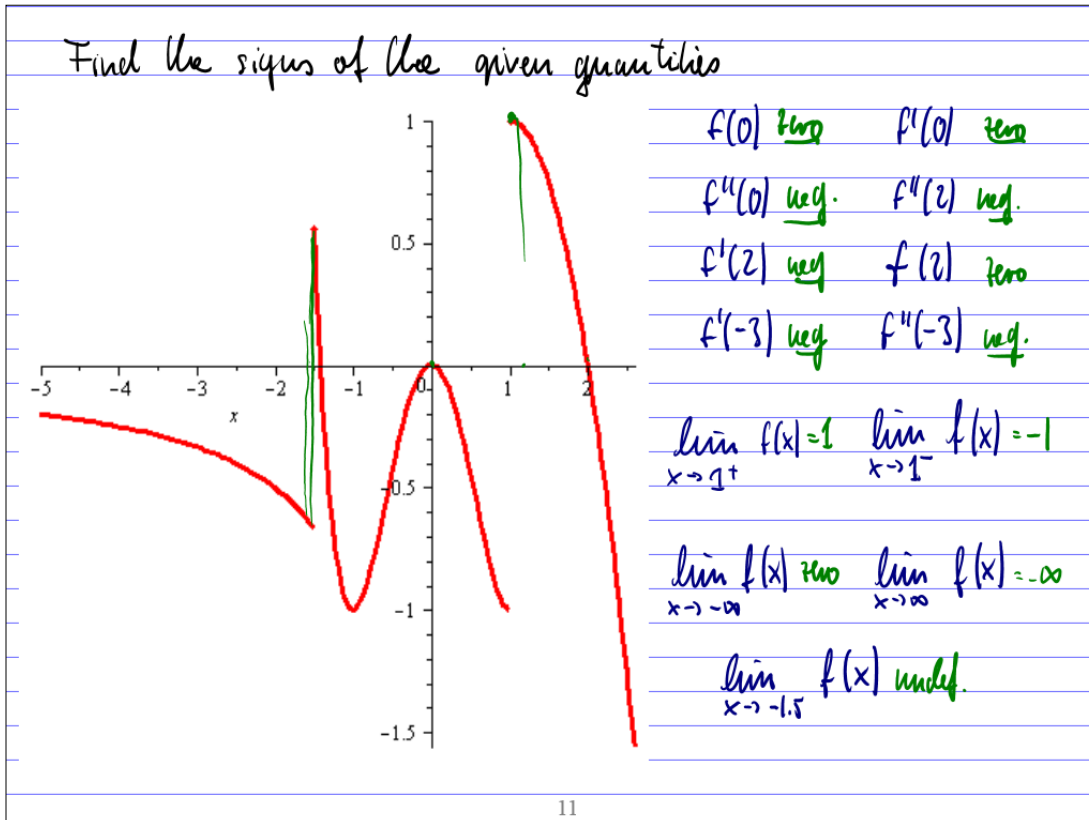
② $f(-2) = 12$
 $f(-1/2) = 7/2$
 $f(1) = -10$

② $f''(x) = 12x + 6 = 0$
 $x = -1/2$



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Panel 11



Panel 12

Ex. $f(x) = 4x - e^x$. Find (a) extrema and (b) concavity

HW

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Panel 13

Graph $f(x) = 3x^4 - 4x^3$

① $f'(x) = 12x^3 - 12x^2 = 12x^2(x-1) = 0$, $x = 1, 0$

② $f''(x) = 36x^2 - 24x = 12x(3x-2) = 0$, $x = \frac{2}{3}, 0$

	-1	0	$\frac{2}{3}$	1	1000000
f'	-	-	-	+	
f''	+	-	+	+	
f					

③ $f(0) = 0$

$f(\frac{2}{3}) = -0.678$

$f(1) = -1$

