Panel 1

Last time: Max / Mins

① $f'(x)$

② $f'(x) = 0$ or $f'(x)$ does not exist

$\Rightarrow$ critical points

③

	CP	CP	
f'	+	−	+
f	↗	↘	↗

④ Read off the answer

1

Panel 2

First vs Second Derivative

First deriv.
- incr. / decr.
- max / mins
- rate of change

Second deriv = rate of change of f'

Ex: s is distance function of time

$\Rightarrow s' = \text{velocity} = v(t) = s'(t)$

$s'' = \text{acceleration} = a(t) = v'(t) = s''(t)$

2

Panel 3

Ex: Suppose $s(t) = -16t^2 + 10t + 5$ is the distance function for some object. Find

a) velocity and acceleration after 10 seconds

$v(t) = s'(t) = -32t + 10 \Rightarrow v(10) = -320 + 10 = -310$

$a(t) = v'(t) = -32$

b) When is the velocity zero? Interprete!

$v(t) = -32t + 10 = 0 \Rightarrow \boxed{t = \frac{10}{32}}$

gives top of the parabola, highest point, or max

3

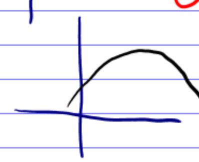
Panel 4

Meaning of f''

$f'' > 0$ means f' is increasing

concave up

$f'' < 0$ means f' is decreasing



concave down

$f'' = 0$ possible inflection point

Ex: $f(x) = 2x^3 - 6x^2$ investigate concavity

$f'(x) = 6x^2 - 12x$

$f''(x) = 12x - 12 = 0 \Rightarrow x = 1$

is possible inflect. point

	1	
	0	2
f''	$-$	$+$
f	$\frown$	$\smile$

4

Panel 5

Questions about increasing / decreasing / max / min

1. $f'(x)$
2. Solve $f'(x) = 0$
3. Make table with signs of f'

Questions about concavity / inflection points

1. $f''(x)$
2. Solve $f''(x) = 0$
3. Make a table with signs of f''

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Panel 6

Ex: $f(x) = x^2 - 50 \cdot \ln(x)$, $\underline{x > 1}$

① Find interval(s) where f is increasing : $(5, \infty)$ $f'(x) = 2x - 50x^{-1}$

$f'(x) = 2x - \frac{50}{x} = 0$

$2x = \frac{50}{x}$

$x^2 = 25$

$x = \pm 5$ is critical

		5 $\Downarrow$
f'	$-$	$+$
f	$\searrow$	$\nearrow$

② Intervals where f is concave up. : $(1, \infty)$

$f''(x) = 2 + \frac{50}{x^2} = 0$

$\frac{50}{x^2} = -2$

$-25 = x^2$

no solution

f''	$+$
f	$\cup$

6

Panel 7

Recipe for Curve Sketching : $f(x) = x^3 - 9x^2 + 15x - 4$

① Find f'

Solve $f' = 0$

① $f'(x) = 3x^2 - 18x + 15 = 3(x^2 - 6x + 5) = 3(x-5)(x-1) = 0$

critical points : $x = 1, 5$

② Find f''

Solve $f'' = 0$

② $f''(x) = 6x - 18 = 0$

possible infl. points : $x = 3$

③

f'			
f''			
f			

③ 0 1 2 3 4 5 6

f'	+	−	−	+
f''	−	−	+	+
f				

④ Find $f(x)$ at special points.

④ $f(1) = 3$

$f(3) = -13$

$f(5) = -29$

⑤ Draw function

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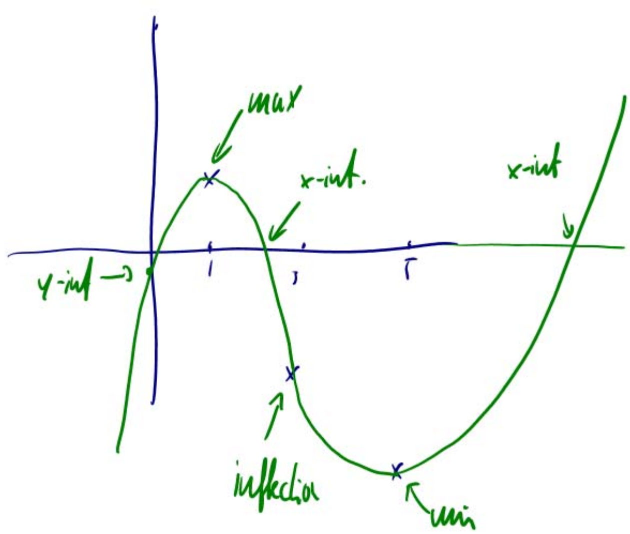
Panel 8



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Panel 9

Carefully sketch the graph of $y = 2x^3 + 3x^2 - 12x - 3$. Identify all critical points and points of inflection. State the intervals over which the graph is increasing, decreasing. Concave up and concave down. Identify any absolute or relative extrema. (8%).

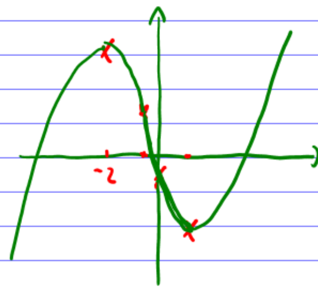
(1) $f'(x) = 6x^2 + 6x - 12 =$
$= 6(x^2 + x - 2) =$
$= 6(x+2)(x-1) = 0$
$x = -2, 1$ are critical

(2) $f''(x) = 12x + 6 = 0$
$x = -1/2$

(3)

		-2	$-1/2$	1
f'	$+$	$-$	$-$	$+$
f''	$-$	$-$	$+$	$+$
f				

(4) $f(-2) = 17$
$f(-1/2) = 7/2$
$f(1) = -10$
y-int: $f(0) = -3$



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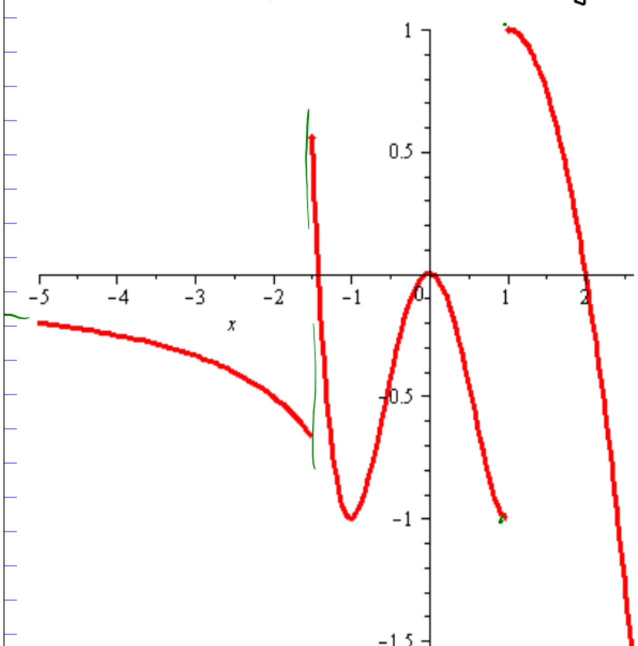
Panel 10

Find the signs of the given quantities



$f(0)$ zero $\quad f'(0)$ zero
$f''(0)$ neg $\quad f''(2)$ neg.
$f'(2)$ neg $\quad f(2)$ zero
$f'(-3)$ neg $\quad f''(-3)$ neg

$\lim_{x \to 1^+} f(x) = 1 \quad \lim_{x \to 1^-} f(x) = -1$

$\lim_{x \to -\infty} f(x)$ zero $\quad \lim_{x \to \infty} f(x) = -\infty$

$\lim_{x \to -1.5} f(x)$ undef.

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Panel 11

Ex: $f(x) = 4x - e^x$. Find (a) extrema and (b) concavity

HW

11

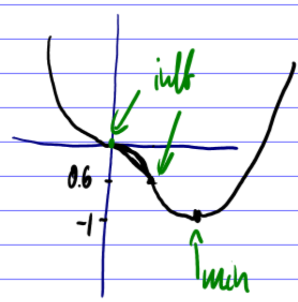
Panel 12

Graph $f(x) = 3x^4 - 4x^3$ $\qquad f(1) = -1$

$f'(x) = 12x^3 - 12x^2 = 12x^2(x-1) \qquad x=1, x=0 \qquad f(0) = 0$

$f''(x) = 36x^2 - 24x = 12x(3x-2) \qquad x = 2/3, x = 0 \qquad f(2/3) = 0.6$

	0	2/3	1	
f'	−	−	−	+
f''	+	−	+	+
f				



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