Panel 1

Different Terms for "Derivative":

Definition $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

marginal cost, revenue, profit

differentiate

slope of tangent

rate of change

velocity (physics)

increasing or decreasing

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Panel 2

Ex: $C(x) = 5\ln(x) + 7e^x - 5x^4 + 2\overset{x^{1/2}}{\sqrt{x}} + \ln(\pi) + 5 + \sqrt{2} + e^3$

Find the rate of change of the marginal cost function.

Marginal cost: $C'(x) = \overset{x^{-1}}{\frac{5}{x}} + 7e^x - 20x^3 + x^{-1/2}$

rate of change of marginal cost:

$$C''(x) = -5x^{-2} + 7e^x - 60x^2 - \frac{1}{2}x^{-3/2}$$

$$\frac{5}{x} = 5x^{-1} \Rightarrow \frac{d}{dx}(5x^{-1}) = 5 \cdot (-1)x^{-2} = -5x^{-2}$$

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Panel 3

Ex: If $c(q) = 0.2q + 2 + \frac{500}{q}$ is the average cost then find. i.e. $c(q) = \frac{C(q)}{q}$

a) $C(q) = c(q)\cdot q = 0.2q^2 + 2q + 500$

b) Fixed cost $C(0) = 500$

c) Marginal cost: $C'(q) = 0.4q + 2$

$C'(1) = 2.4$

d) Rate of change of marginal cost

$C''(0.4)$

$\frac{d}{dx} 7x^3 = 21x^2$

$\frac{d}{dx} 3\sqrt{x} = \frac{d}{dx} 3x^{1/2} = 3\frac{1}{2}x^{-1/2}$

$\frac{d}{dx}\left(\frac{7}{x^3}\right) = \frac{d}{dx}(7x^{-3}) = -21x^{-4}$

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Panel 4

Name: ________

Quiz #7

① If $P(x) = 10\ln(x) + e^x$ is a profit function

a) find the marginal profit for $x = 1$

$P'(x) = 10\cdot\frac{1}{x} + e^x \Rightarrow P'(x=1) = \frac{10}{1} + e^1 > 0$

b) Should you increase or decrease production from its current level of $x = 1$?

increase

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Panel 5

(2) Find the indicated derivative for the function:

a) $f(x) = 3x^2 - 2\sqrt{x} + 3 \cdot \ln(x)$; find $f''(x)$

$x^{1/2}$

$f'(x) = 6x - 2\frac{1}{2}x^{-1/2} + 3 \cdot \frac{1}{x} = 6x - x^{-1/2} + 3x^{-1}$

$f''(x) = 6 - (-\frac{1}{2})x^{-3/2} + 3(-1)x^{-2}$

b) $f(x) = 4x^3 - 3x^2 + 7x - 9$; find $f^{(10)}(x)$, the 10^{th}-deriv

$f'(x) = 12x^2 - 6x + 7$

$f''(x) = 24x - 6$

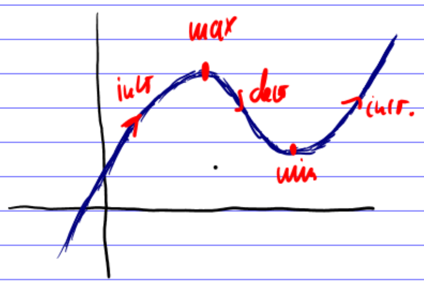
$f'''(x) = 24$

$f^{(4)}(x) = 0$

$\boxed{\Rightarrow f^{(10)}(x) = 0}$

Panel 6

More appl. of derivative: Max / Min of a Function



$f' > 0$ means

f is increasing

$f' < 0$ means

f is decreasing

Thm: If f has a local max. or min. at $x = c$ then either $f'(c) = 0$ or $f'(c)$ does not exist.

Panel 7

How to Find local Max / Min

Ex: $f(x) = 2x^3 + 3x^2 - 12x - 3$ Find all local extrema

$f'(x) = 6x^2 + 6x - 12$

critical points

$0 = 6x^2 + 6x - 12$

$= 6(x^2 + x - 2) = 6(x+2)(x-1) \Rightarrow x = -2, 1$

	-3	-2	0	1	2
f'	$+$		$-$		$+$
f	↗		↘		↗

$x=-2$ is max

$x=1$ is a min

Recipe

1. Find $f'(x)$
2. Find critical points, i.e. where $f'(x) = 0$ or $f'(x)$ does not exist
3. Make a table of f, f' and figure the sign of f' between all critical points
4. Read off answer

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Panel 8

Ex: $f(x) = x^2 + 6x - 8$ Find local max / min

$f'(x) = 2x + 6$

$0 = 2x + 6 \Rightarrow x = -3$ is critical point   is obvious since f is a parabola

		-3	
f'	$-$		$+$
f	↘		↗

$x=-3$ has a min.

$f(x) = ax^2 + bx + c$

$f'(x) = 2ax + b = 0$

$x = -b/2a$

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Panel 9

Ex: Suppose $C(x) = \frac{360000}{x} + 4x$ is a cost function based on the inventory $x \geq 0$. How much inventory should you carry to minimize the cost?

$C(x) = 360000x^{-1} + 4x$

$C'(x) = -360000x^{-2} + 4 = -\frac{360000}{x^2} + 4$

$-\frac{360000}{x^2} + 4 = 0 \Rightarrow +\frac{360000}{x^2} = +4 \Leftrightarrow \frac{360000}{4} = x^2 \Leftrightarrow 90000 = x^2 \quad |\sqrt{\ }$

$\Rightarrow \pm \boxed{300} = x$

	1	300	Billions
C'	$-$		$+$
C	↘		↗

Thus, $x = 300$ gives a minimum, and the minimum cost is $C(300) = \frac{360000}{300} + 4 \cdot 300$.

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