Panel 1

The (Calc) Story so far....

$$\lim_{x\to a} f(x) = L$$ as x gets closer to a, f(x) gets closer to L

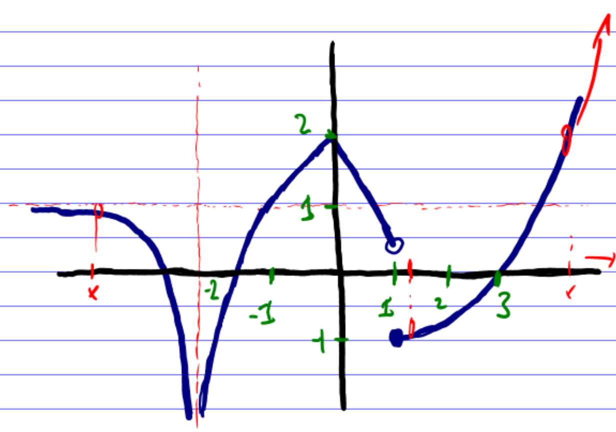
f continuous at $x=a$ $\left\{ \begin{matrix} f(a) \\ \lim_{x\to a} f(x) \end{matrix} \right\}$ same?

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$ differentiable means f' exists

slope of tangent, $\frac{d}{dx}$, inst. rate of change, velocity, marginal $\left\{ \begin{matrix} \text{cost} \\ \text{revenue} \\ \text{profit} \end{matrix} \right.$

1

Panel 2



$f(3) = 0$

$f'(3)$ pos

$$\lim_{x\to 1^+} f(x) = -1$$

$$\lim_{x\to -\infty} f(x) = 1$$

List points where f is not continuous $x=-2$ and $x=1$

List points where f is not differentiable $x=0$, $x=-2$, $x=1$

Note: If f is differentiable, it must be continuous.

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Panel 3

Differentiation Shortcuts

too messy

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

too imprecise

f' is the slope of the tangent line

Product Rule: $\frac{d}{dx}\left[x^n\right] = nx^{n-1}$

Sum Rule: $\frac{d}{dx}\left(f(x) \pm g(x)\right) = f'(x) \pm g'(x)$

Constant Factor: $\frac{d}{dx}\left(c \cdot f(x)\right) = c\, f'(x)$, e.g. $\frac{d}{dx}\left[5x^3\right] = 5 \cdot 3x^2 = 15x^2$

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Panel 4

$$f(x) = \begin{cases} \frac{x^2-9}{x-3} & \text{if } x \neq 3 \\ \cancel{15}\ 6 & \text{if } x = 3 \end{cases}$$

cont. at $x = 3$?

(1) $f(3) = \cancel{15}\ 6$

(2) $\lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{x^2-9}{x-3} = \lim_{x \to 3} \frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}} = 6$

(1) = (2) YES

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Panel 5

17) slope $y=4$ at $x=-4, 7, 22$

$y'=0$

$x^{8/7}$ x^{-4}

$y = 5\sqrt[7]{x^8} - \frac{5}{x^4}$

$y' = 5\frac{8}{7}x^{1/7} - 5(-4)x^{-5}$

$\frac{40}{7}x^{1/7} + 20x^{-5}$

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Panel 6

Name: ______

Quiz: Find the following derivatives:

a) $f(x) = x^3 \Rightarrow f'(x) =$

b) $f(x) = \sqrt[4]{x} \Rightarrow f'(x)$

c) $f(x) = 5x^2 - 6x + 1 \Rightarrow f'(x)$

d) $\frac{d}{dx}\left[4x^3 - 5\sqrt{x^3} + \frac{5}{x^2} + e^2\right]$

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Panel 7

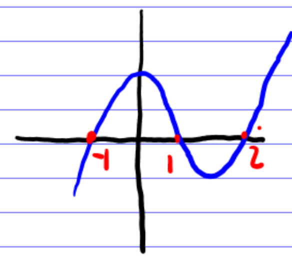
(2) For which value of k, if any, is the following function continuous at $x = 2$?

$$f(x) = \begin{cases} \frac{x-2}{x^2-4} & \text{if } k \neq 2 \\ k & \text{if } k = 2 \end{cases}$$

(3) Is $f'(x)$ positive, negative, or zero?

a) $f'(-1)$

b) $f'(0)$



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Panel 8

Name: ______

Quiz: Find the following derivatives:

a) $f(x) = x^4 \Rightarrow f'(x) =$

b) $f(x) = \sqrt[5]{x} \Rightarrow f'(x)$

c) $f(x) = 4x^2 - 3x + 1 \Rightarrow f'(x)$

d) $\frac{d}{dx}\left[4x^5 - 2\sqrt{x^3} + \frac{3}{x^2} + e^2\right]$

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Panel 9

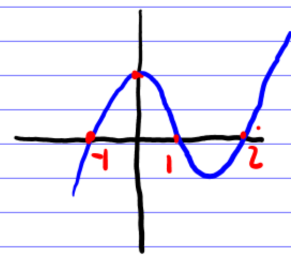
(2) For which value of k, if any, is the following function continuous at $x = 2$?

$$f(x) = \begin{cases} \dfrac{x-2}{x^2-4} & \text{if } k \neq 2 \\ k & \text{if } k = 2 \end{cases}$$

(3) Is $f'(x)$ positive, negative, or zero?

a) $f'(0)$

b) $f'(1)$



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Panel 10

Ex: A sociologist believes that x years after the beginning of a program, $f(x)$ - thousand preschoolers will be enrolled, where

$$f(x) = \frac{10}{9}(12x - x^2) \quad , 0 \le x \le 12$$

Suppose the program is in existence for 9 years. How much increase in enrollment is to be expected?

rate of change?

$f'(x) = \frac{10}{9}(12 - 2x)$ thus $f'(9) = \frac{10}{9}(12 - 18) = -\frac{60}{9} = -6.666...$

Enrollment will decrease by 6.666 thou. students

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Panel 11

Marginal Cost: The marginal cost is the approx. cost of producing one additional unit.

$C'(x)$ is marginal cost

Ex: A cost function is $C(q) = 0.0001q^3 - 0.02q + 5000$

Find fixed cost and marginal cost when $q = 50$

Fixed cost: $C(0) = 5000$

Marginal cost: $C'(q) = 0.0003q^2 - 0.02$ at $q = 50$

$C'(50) = 0.73$

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Panel 12

Relative Rate of Change

If $f(x)$ is a function, rate of change is

The rate of change relative to $f(x)$ is called

the relative rate of change $\frac{f'(x)}{f(x)}$

Ex: Find relative rate of change for $y = 3x^2 - 5x + 25$ when $x = 5$.

$y' = 6x - 5 \Rightarrow y'(5) = 30 - 5 = 25$

$y(5) = 3 \cdot 25 - 25 + 25 = 75$

$\Rightarrow$ rel. rate of change $\frac{25}{75} = \frac{1}{3}$

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Panel 13

Final example: Suppose a revenue function is

$$R(q) = \frac{2q^2+20}{q+1}.$$ For what level of production is the marginal revenue zero?

$R'(q) = 0$

Results: Show steps | More digits

discard

$q = -1-\sqrt{11} \approx -4.31662$

$q = \sqrt{11} - 1 \approx 2.31662$

solve (2 (-10 + 2 q + q^2))/(1 + q)^2=0 WolframAlpha

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Panel 14

Derivatives of special functions

If $f(x) = e^x$ then $f'(x) = e^x$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{e^{x+h}-e^x}{h} =$$

$$= \lim_{h\to 0} \frac{e^x e^h - e^x}{h} = \lim_{h \to 0} e^x \left[\frac{e^h - 1}{h}\right] = \boxed{e^x}$$

Power Rule does not apply!!!

h	$\frac{e^h-1}{h}$
0.0001	1.0000500..
⋮	⋮
	1

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Panel 15

Derivative of e^x and $\ln(x)$:

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} \ln(x) = 1/x$$

Ex: $f(x) = e^2 + e^x + x^2 + x^e + \ln(x) + \frac{1}{x} + x + 1$ ($\frac{1}{x} = x^{-1}$)

$f'(x) = 0 + e^x + 2x + ex^{e-1} + \frac{1}{x} - x^{-2} + 1 + 0$

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Panel 16

Ex: A cost function is given by

$$C(q) = 25 \cdot \ln(q) + q^2, \quad q \geq 1$$

Find the marginal cost when $q = 6$ and interpret it.

$C'(q) = \frac{25}{q} + 2q$ so at $q = 6$

$C'(6) = \frac{25}{6} + 12 > 0$ so

cost per additional item is positive ~16

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Panel 17

Higher Order Derivatives

If $f(x)$ is a (differentiable) function, then $f'(x)$ is again a function.

$f(x) \Rightarrow f'(x)$ is 1^{st}-derivative $= \frac{d}{dx} f$

$f'(x) \Rightarrow f''(x)$ is 2^{nd}-deriv. $= \frac{d^2}{dx^2} f$

$f''(x) \Rightarrow f'''(x)$ is 3^{rd} deriv $= \frac{d^3}{dx^3} f$ ...

$\vdots$

$f^{(n)}(x)$ is n-th derivative

or $\frac{d^n}{dx^n} f = f^{(n)}(x)$

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Panel 18

Ex: If $f(x) = 3x^4 - 9x^2 + 5x$, find f''

$f'(x) = 12x^3 - 18x + 5$

$f''(x) = 36x^2 - 18$

Ex: $f(x) = 5e^x - 7\ln(x) + \sqrt{x}$. Find f'''

$\sqrt{x} = x^{1/2}$

$f'(x) = 5e^x - 7/x + \frac{1}{2}x^{-1/2}$

$-7/x = -7x^{-1}$

$f''(x) = 5e^x + 7x^{-2} - \frac{1}{4}x^{-3/2}$

$f'''(x) = 5e^x - 14x^{-3} + \frac{1}{4} \cdot \frac{3}{2} \cdot x^{-5/2}$

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Panel 19

Sometimes higher order derivatives become easy:

Ex: $f(x) = 3x^3 - 2x^2 + x$. Then

$f'(x) = 9x^2 - 4x + 1$

$f''(x) = 18x - 4$

$f'''(x) = 18$

$f''''(x) = f^{(4)}(x) = 0$

$f'''''(x) = f^{(5)}(x) = 0$

$\vdots$

$f^{(100)}(x) = 0$

Have a nice Spring break!

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Panel 20

Interpretation:

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