Panel 1

The (Calc) Story so far...

$\lim_{x \to a} f(x) = L$ as x gets closer to a, $f(x)$ gets closer to L

f continuous at $x = a$ 1) $f(a)$ 2.) $\lim_{x \to a} f(x)$ } same?

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{x \to a} \cancel{\frac{f(x) - f(a)}{x - a}}$ exists if f is differentiable

$\frac{d}{dx}$, derivative, slope of tangent, inst. rate of change, velocity, marginal cost revenue profit

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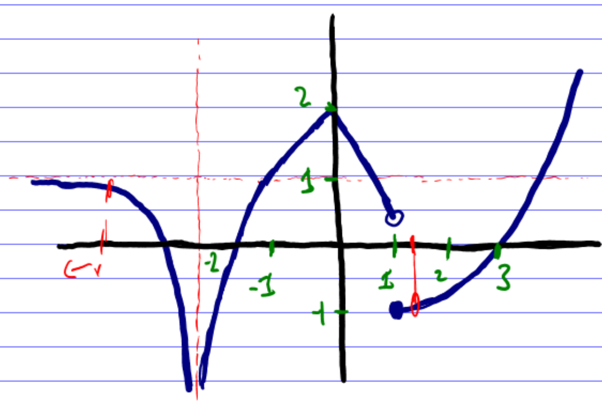
Panel 2



$f(3) = 0$

$f'(3)$ pos.

$\lim_{x \to 1^+} f(x) = -1$

$\lim_{x \to -\infty} f(x) = 1$

List points where f is not continuous at $x = 1$ and $x = -2$

List points where f is not differentiable $x = 0, x = 1, x = -2$

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Panel 3

Differentiation Shortcuts

too many

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

too imprecise

f' is the slope of the tangent line

Product Rule: $\frac{d}{dx}\left[x^n\right] = n x^{n-1}$

Sum Rule: $\frac{d}{dx}\left(f(x) \pm g(x)\right) = f'(x) \pm g'(x)$

Constant Factor: $\frac{d}{dx}\left(c \cdot f(x)\right) = c\, f'(x)$ e.g. $\frac{d}{dx}\left[5x^3\right] = 5 \cdot 3x^2$

$= 15x^2$

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Panel 4

Name: ________

Quiz: Find the following derivatives:

a) $f(x) = x^3 \Rightarrow f'(x) =$

b) $f(x) = \sqrt[4]{x} \Rightarrow f'(x)$

c) $f(x) = 5x^2 - 6x + 1 \Rightarrow f'(x)$

d) $\frac{d}{dx}\left[4x^3 - 5\sqrt{x^3} + \frac{5}{x^2} + e^2\right]$

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Panel 5

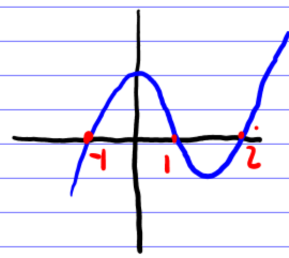
(2) For which value of k, if any, is the following function continuous at $x = 2$?

$$f(x) = \begin{cases} \dfrac{x-2}{x^2-4} & \text{if } k \neq 2 \\ k & \text{if } k = 2 \end{cases}$$

(3) Is $f'(x)$ positive, negative, or zero?

a) $f'(-1)$

b) $f'(0)$



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Panel 6

Name: ________

Quiz: Find the following derivatives:

a) $f(x) = x^4 \Rightarrow f'(x) = 4x^3$

b) $f(x) = \sqrt[5]{x} \Rightarrow f'(x)\ \frac{1}{5}x^{-4/5} = \frac{1}{5x^{4/5}}$

c) $f(x) = 4x^2 - 3x + 1 \Rightarrow f'(x)\ \ 8x - 3$

d) $\frac{d}{dx}\left[4x^5 - 2\sqrt{x^3} + \frac{3}{x^2} + \cancel{e^2}\right]$

$20x^4 - 2\cdot\frac{3}{2}x^{1/2} - 6x^{-3}$

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Panel 7

(2) For which value of k, if any, is the following function continuous at $x = 2$?

$$f(x) = \begin{cases} \frac{x-2}{x^2-4} & \text{if } k \neq 2 \\ k & \text{if } k = 2 \end{cases}$$

1/4

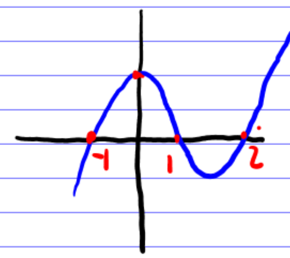
$$\lim_{x \to 2} \frac{\cancel{x-2}}{(x+2)(\cancel{x-2})} \quad \frac{1}{4}$$

(3) Is $f'(x)$ positive, negative, or zero?

a) $f'(0)$ zero

$f'(-1)$ pos

b) $f'(1)$ neg.



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Panel 8

Ex: A sociologist believes that x years after the beginning of a program, f(x) - thousand preschoolers will be enrolled, where

$$f(x) = \frac{10}{9}(12x - x^2), \quad 0 \le x \le 12$$

Suppose the program is in existence for 9 years. How much increase in enrollment is to be expected?

what is the rate of change, i.e. $f'(x)$

$$f'(x) = \frac{10}{9}(12 - 2x), \quad f'(9) = \frac{10}{9}(12 - 18) = -\frac{60}{9} = -6.666\ldots \text{ thousand}$$

The is an expected decrease in enrollment of 6.666... students

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Panel 9

Marginal Cost: The marginal cost is the approx. cost of producing one additional unit.

marginal cost is $C'(x)$

Ex: A cost function is $C(q) = 0.0001q^3 - 0.02q + 5000$

Find fixed cost and marginal cost when $q = 50$

Fixed cost is $C(0) = 5000$

Marginal cost is $C'(q) = 0.0003q^2 - 0.02$ so at $q = 50$

$C'(50) = 0.055$

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Panel 10

Relative Rate of Change

If $f(x)$ is a function, rate of change is

The rate of change relative to $f(x)$ is called the relative rate of change $\frac{f'(x)}{f(x)}$

Ex: Find relative rate of change for $y = 3x^2 - 5x + 25$ when $x = 5$.

$y'(x) = 6x - 5$, $y'(5) = 30 - 5 = 25$

$y(5) = 3 \cdot 25 - 25 + 25 = 75$

Thus, relative rate of change is $\frac{25}{75} = \frac{1}{3}$

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Panel 11

Final example: Suppose a revenue function is

$$R(q) = \frac{2q^2+20}{q+1}$$

. For what level of production is the marginal revenue zero?

Marginal ___ = derivative so

$R'(q)$

discard

Results: Show steps | More digits

$q = -1-\sqrt{11} \approx -4.31662$

$q = \sqrt{11}-1 \approx 2.31662$

solve (2 (−10 + 2 q + q^2))/(1 + q)^2=0 WolframAlpha

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Panel 12

Derivatives of special functions

If $f(x) = e^x$ then $f'(x) = e^x$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{e^{x+h}-e^x}{h} =$$

$$= \lim_{h\to 0} \frac{e^x e^h - e^x}{h} = \lim_{h\to 0} e^x \left(\frac{e^h-1}{h}\right) = e^x$$

1

power rule does NOT apply!

h	$\frac{e^h-1}{h}$
0.0001	1.000500...
	⋮
	1

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Panel 13

Derivative of e^x and $\ln(x)$:

$$\left.\begin{aligned} \frac{d}{dx} e^x &= e^x \\ \frac{d}{dx} \ln|x| &= 1/x \end{aligned}\right\} \quad \underline{\text{Memorize!}}$$

Ex: $f(x) = e^2 + e^x + \underline{x^2} + \underline{x^e} + \ln(x) + \frac{1}{x} + x + 1$

$\frac{1}{x} = x^{-1}$

$$f'(x) = 0 + e^x + 2x + ex^{e-1} + \frac{1}{x} - x^{-2} + 1 + 0$$

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Panel 14

Ex: A cost function is given by

$$C(q) = 25 \ln(q) + q^2 \quad , q \geq 1$$

Find the marginal cost when $q = 6$ and interpret it.

$C'(q) = 25 \cdot \frac{1}{q} + 2q$ at $q = 6$:

$$C'(6) = \frac{25}{6} + 12 > 0$$

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Panel 15

Higher Order Derivatives

If $f(x)$ is a (differentiable) function, then $f'(x)$ is again a function.

$f(x) \Rightarrow f'(x)$ is 1st derivative

$f'(x) \Rightarrow f''(x)$ is 2nd derivative $\frac{d^2}{dx^2} f$

$f''(x) \Rightarrow f'''(x)$ is 3rd derivative

$\vdots \qquad \vdots$

$f^{(n)}(x)$ is n-th derivative

or: $\frac{d^n}{dx^n} f = f^{(n)}(x)$

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Panel 16

Ex: If $f(x) = 3x^4 - 9x^2 + 5x$, find f''

$f'(x) = 12x^3 - 18x + 5$

$f''(x) = 36x^2 - 18$

Ex: $f(x) = 5e^x - 7\ln(x) + \sqrt{x}$. Find $\frac{d^3}{dx^3} f$

$f'(x) = 5e^x - \frac{7}{x} + \frac{1}{2} x^{-1/2}$ ($7x^{-1}$)

$f''(x) = 5e^x + 7x^{-2} - \frac{1}{4} x^{-3/2}$

$f'''(x) = 5e^x - 14x^{-3} + \frac{3}{8} x^{-5/2}$

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Panel 17

Sometimes higher order derivatives become easy:

Ex: $f(x) = 3x^3 - 2x^2 + x$. Then

$f'(x) = 9x^2 - 4x + 1$

$f''(x) = 18x - 4$

$f'''(x) = 18$

$f''''(x) = f^{(4)}(x) = 0$

$f'''''(x) = f^{(5)}(x) = 0$

$\vdots$

$f^{(100)}(x) = 0$

$f(x) =$

Have a nice Break!

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