Panel 1

Last Time:

$\lim_{x \to a} f(x) = L$ means:

$\lim_{x \to a^+} f(x)$ x is close to a, but $x > a$

$\lim_{x \to a^-} f(x)$ x close to a, but $x < a$

$\lim_{x \to \pm\infty} f(x)$ $\begin{cases} \deg(\text{top}) = \deg(\text{bottom}) \Rightarrow \#/\# \\ \deg(\text{top}) < \deg(\text{bottom}) \Rightarrow 0 \\ \deg(\text{top}) > \deg(\text{bottom}) \Rightarrow \text{undef.} \end{cases}$

1

Panel 2

Ex: Find the following limits:

$\lim_{x \to 0} \frac{x^2 - 2x + 1}{x^2 - 4} = -1/4$

$\lim_{x \to 2} \frac{x^2 + 2x - 8}{x^2 - 4} = \frac{0}{0} = \lim_{x \to 2} \frac{(x+4)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}} = 6/4 = 3/2$

$\lim_{x \to 1^+} f(x) = 3$ where $f(x) = \begin{cases} x^2 - 1 & \text{if } x < 1 \\ 3x & \text{if } x > 1 \end{cases}$

$\lim_{x \to 1^-} f(x) = 0$

$\lim_{x \to 1} f(x) = \text{undef.}$ $\lim_{x \to 2} f(x) = 3$

$\lim_{x \to \infty} \frac{3x^3 - 2x + 9}{1 - 2x - 3x^2 - 4x^3} = 3/-4 = -3/4$

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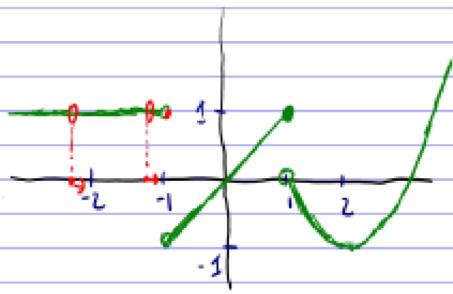
Panel 3

Ex: Consider the graph of the function shown below



a) $\lim_{x \to -2} f(x) = 1$

b) $\lim_{x \to 0^+} f(x) = 0$

c) $\lim_{x \to -1^-} f(x) = 1$

d) $\lim_{x \to 1^+} f(x) = 0$

e) $\lim_{x \to 0} f(x) = 0$

f) $\lim_{x \to -1} f(x) =$ ☹

g) $\lim_{x \to 1} f(x)$ dne or undefined or ☹

$\lim_{x \to 1^-} f(x) = 1$

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Panel 4

Quiz #4

Name: ______

① Find the following limits

a) $\lim_{x \to 0} \frac{x^2 - 5x + 4}{x - 2} = 4/_{-2} = -2$

b) $\lim_{x \to 2} \frac{x^2 - 3x + 2}{x^2 - 4} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x-1)}{(x+2)\cancel{(x-2)}} = 1/4$

c) $\lim_{x \to \infty} \frac{6x^2 - 5x + 2}{7 - 3x^2} = \frac{6}{-3} = -2$

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Panel 5

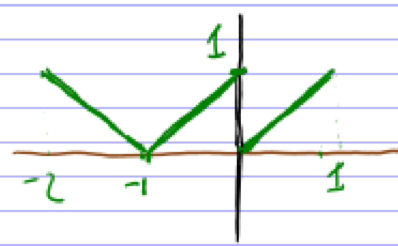
(2) Suppose $f(x) = \begin{cases} x^2+3 & \text{if } x>0 \\ 3x-1 & \text{if } x<0 \end{cases}$ Then

find $\lim_{x\to 0} f(x)$ if possible undefined

$\lim_{x\to 0^+} f(x) = +3$

$\lim_{x\to 0^-} f(x) = -1$

(3) For f as shown, find



a) $\lim_{x\to 0^-} f(x) = 1$

b) $\lim_{x\to -1^+} f(x) = 0$

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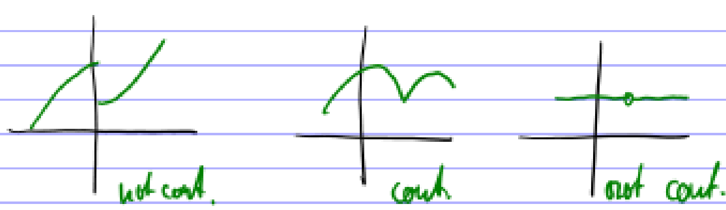
Panel 6

Continuity

Most simple functions have a graph that you can draw without lifting up the pen. ⇒ continuous



Def: f is continuous at $x=a$ if

(1) $f(a)$ is defined

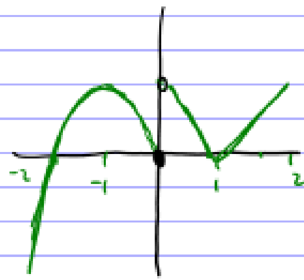
(2) $\lim_{x\to a} f(x)$ is defined

(3) (1) = (2)

6

Panel 7

Ex:



$\lim_{x \to -1} f(x)$

$\lim_{x \to 0} f(x)$

$\lim_{x \to 1} f(x)$

Is f continuous at:

$x = -1$

$x = 0$

$x = 1$

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Panel 8

Ex: $f(x) = \begin{cases} 1 & \text{if } x < 0 \\ -1 & \text{if } x = 0 \\ x^2 & \text{if } x > 0 \end{cases}$ Is f cont. at $x = 0$?

NO

① $f(0) = -1$

$\lim_{x \to 0^+} f(x) = 0$

② $\lim_{x \to 0} f(x) =$ undef.

$\lim_{x \to 0^-} f(x) = 1$

③ are ① = ②

$f(x) = \begin{cases} x+6 & \text{if } x \geq 3 \\ x^2 & \text{if } x < 3 \end{cases}$ cont. at $x=3$

$f(3) = 9$, $\lim_{x \to 3} f(x) = 9$, $9 = 9$ YES

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Panel 9

Ex:

$$f(x) = \begin{cases} \frac{x^2-9}{x-3} & \text{if } x \neq 3 \\ k & \text{if } x = 3 \end{cases}$$

What is k so that f is cont at x=3

yes if k=6

(1) $f(3) = \underline{k}$

(2) $\lim_{x\to 3} f(x) = \lim_{x\to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3} \frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}} = \underline{6}$

(3) $k = 6$

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Panel 10

Continuity is a property that most functions (defined in one line) automatically have.

Every polynomial is continuous

Every rational function is continuous where denom is not zero

$\lim_{x\to 1} 5x-1 = 4$

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Panel 11

Applications of Limits

Ex: The population of a small city is $P(t) = 50{,}000 - \frac{2000}{t+1}$.

a) What is the current population $P(0) = 48000$

b) What is the population in the long run?

$$\lim_{t\to\infty} P(t) = 50000 - 0 = 50000$$

Ex: For a host-parasite relation it was determined that when the host density is x (# of hosts per area) the number of host parasites is $y = \frac{900x}{10+45x}$ $\lim_{x\to\infty} \frac{900x}{10+45x} = \frac{900}{45} = 20$

If hosts increase without bound, what about parasites?

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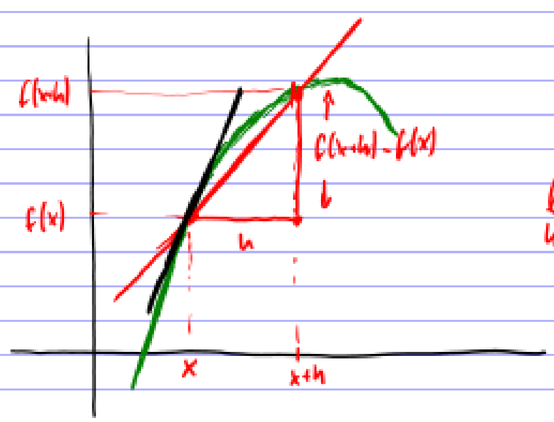
Panel 12

The Most Famous Limit of All:

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \cdot \frac{f(x) - f(x)}{0} = \frac{0}{0}$$



$\frac{f(x+h) - f(x)}{h}$ is slope of the secant line (red)

$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ is slope of tangent line (black)

(just touches graph)

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Panel 13

The Derivative

If $f(x)$ is a function, we define the derivative of f as

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

("f prime")

provided the limit exists.

Geometrically, $f'(x)$ is the slope of tangent line.

Ex: $f(x) = x^2$. Find $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} =$

$$= \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0} \frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}} = \lim_{h\to 0} 2x+h = 2x$$

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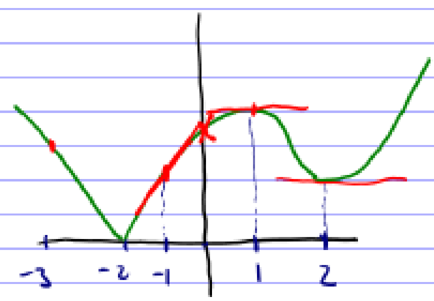
Panel 14

Ex: Find the indicated quantities: (sign only)



$f'(-3)$ neg.

$f'(-1)$ pos

$f'(0)$ pos

$f'(1)$ zero

$f'(2)$ zero

$f'(1.5)$ neg.

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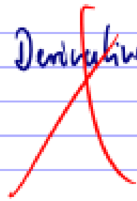
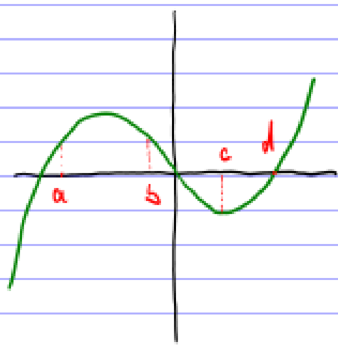
Panel 15

Def: The Derivative of f at a point x is:



Know that...

Ex:



Estimate the signs of

1) $f'(a)$
2) $f'(b)$
3) $f'(0)$
4) $f'(c)$
5) $f'(d)$

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Panel 16

Ex: Find the derivative to $f(x) = 2x^2 - x + 1$ at the point $x = 1$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{[2(x+h)^2 - (x+h) + 1] - [2x^2 - x + 1]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{x} - h + \cancel{1} - \cancel{2x^2} + \cancel{x} - \cancel{1}}{h} =$$

$$= \lim_{h\to 0} \frac{4xh + 2h^2 - h}{h} = \lim_{h\to 0} \frac{h(4x + 2h - 1)}{h}$$

$$= \lim_{h\to 0} 4x + 2h - 1 =$$

$$= 4x - 1$$

$$\boxed{f'(1) = 3}$$

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