Panel 1

Last Time:

$\lim_{x \to a} f(x) = L$ means:

$\lim_{x \to a^+} f(x)$ x close to a, but $x > a$

$\lim_{x \to a^-} f(x)$ x close to a, but $x < a$

$$\lim_{x \to \pm\infty} f(x) \begin{cases} \deg(\text{top}) = \deg(\text{bottom}) & \Rightarrow \#/\# \\ \deg(\text{top}) < \deg.(\text{bottom}) & \Rightarrow 0 \\ \deg(\text{top}) > \deg.(\text{bottom}) & \Rightarrow \text{undef.} \end{cases}$$

1

Panel 2

Ex: Find the following limits:

$\lim_{x \to 1} \frac{x}{x^2-1} = \text{undef.}$

$$\lim_{x \to 0} \frac{x^2-2x+1}{x^2-4} = -1/4$$

$$\lim_{x \to 2} \frac{x^2+2x-8}{x^2-4} = \lim_{x \to 2} \frac{(x+4)\cancel{(x-2)}}{\cancel{(x-2)}(x+2)} = 6/4 = 3/2$$

$$\lim_{x \to 1^+} f(x) = 3 \text{ where } f(x) = \begin{cases} x^2-1 & \text{if } x < 1 \\ 3x & \text{if } x \geq 1 \end{cases}$$

$\lim_{x \to 1^-} f(x) = 0$

$\lim_{x \to -1} f(x) = 0$ $\lim_{x \to 1} f(x) = \text{undef.}$

$$\lim_{x \to \infty} \frac{3x^3-2x+9}{1-2x-3x^2+4x^3} = -3/4$$

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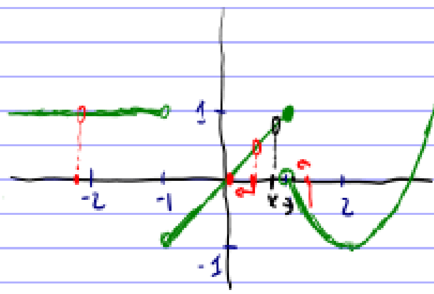
Panel 3

Ex: Consider the graph of the function shown below



a) $\lim_{x \to -2} f(x) = 1$

b) $\lim_{x \to 0^+} f(x) = 0$

c) $\lim_{x \to -1^-} f(x) = 1$

d) $\lim_{x \to 1^+} f(x) = 0$

e) $\lim_{x \to 0} f(x) = 0$

f) $\boxed{\lim_{x \to -1} f(x) = \text{undef}}$

g) $\lim_{x \to 1} f(x)$ undef.

$\lim_{x \to 1^-} f(x) = 1$

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Panel 4

Name: ______

Quiz #4

① Find the following limits

a) $\lim_{x \to 0} \frac{\cancel{x}^2 - 5\cancel{x} + 4}{\cancel{x} - 2} = 4/-2 = -2$

b) $\lim_{x \to 2} \frac{x^2 - 3x + 2}{x^2 - 4} = \frac{0}{0} = \lim_{x \to 2} \frac{(x-2)(x-1)}{(x+2)(x-2)} = \frac{1}{4}$

c) $\lim_{x \to \infty} \frac{6x^2 - 5x + 2}{7 - 3x^2} = 6/-3 = -2$

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Panel 5

(2) Suppose $f(x) = \begin{cases} x^2+3 & \text{if } x>0 \\ 3x-1 & \text{if } x<0 \end{cases}$ Then

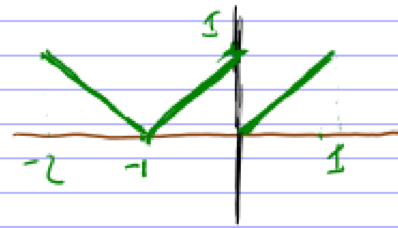
find $\lim_{x \to 0} f(x)$ if possible

undefined!

$\lim_{x \to 0^-} f(x) = -1$

$\lim_{x \to 0^+} f(x) = 3$

(3) For f as shown, find



a) $\lim_{x \to 0^-} f(x) = 1$

b) $\lim_{x \to -1^+} f(x) = 0$

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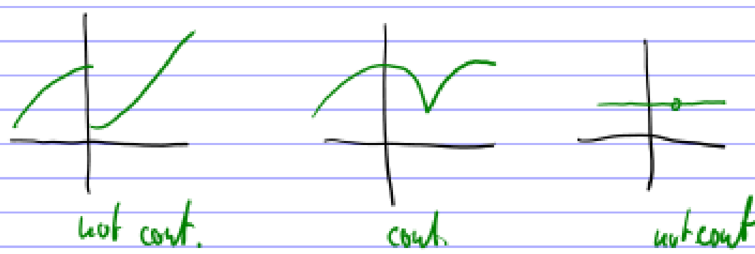
Panel 6

Continuity

Most simple functions have a graph that you can draw without lifting pen → continuous



Def: f is continuous at $x=a$ if

(1) $f(a)$ is defined

(2) $\lim_{x \to a} f(x)$ is defined

(3) (1) = (2)

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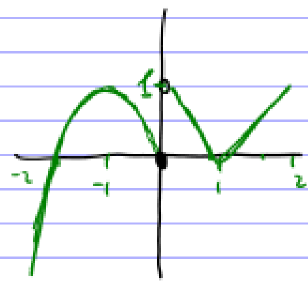
Panel 7

Ex:



$\lim_{x \to -1} f(x) = 1 = f(1)$

$\lim_{x \to 0} f(x)$ undef.

$\lim_{x \to 1} f(x) = 0 = f(1)$

Is f continuous at:

$x = -1$ cont

$x = 0$ not cont

$x = 1$ cont

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Panel 8

Ex: $f(x) = \begin{cases} 1 & \text{if } x < 0 \\ -1 & \text{if } x = 0 \\ x^2 & \text{if } x > 0 \end{cases}$ Is f cont. at $x = 0$?

(1) $f(0) = -1$

(2) $\lim_{x \to 0} f(x) = \text{undef}$ $\left\{ \begin{array}{l} \lim_{x \to 0^-} f(x) = 1 \\ \lim_{x \to 0^+} f(x) = 0 \end{array} \right.$

(3)

not cont. at zero!

$f(x) = \begin{cases} x + 6 & \text{if } x \geq 3 \\ x^2 & \text{if } x < 3 \end{cases}$ cont. at 3. YES

(1) $f(3) = 9$

(2) $\lim_{x \to 3} f(x) = 9$

(3) $(1) = (2)$ ✓

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Panel 9

Ex:

$$f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & \text{if } x \neq 3 \\ K & \text{if } x = 3 \end{cases}$$

Pick k s.t. f(x) is continuous at x = 3

Pick k = 6

① $f(3) = K$

② $\lim_{x\to 3} f(x) = \lim_{x\to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3} \frac{(x+3)\cancel{(x-3)}}{\cancel{x-3}} = 6$

③ is ① = ② $= \boxed{k = 6}$

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Panel 10

Continuity is a property that most functions (defined in one line) automatically have.

Every polynomial is continuous

Every rational function is continuous except where denom = zero

$\lim_{x\to 1} 5x - 1 = 4$

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Panel 11

Applications of Limits

Ex: The population of a small city is $P(t) = 50{,}000 - \frac{2000}{t+1}$.

a) What is the current population $P(0) = 48000$

b) What is the population in the long run?

$$\lim_{t \to \infty} P(t) = 50000 - 0 = 50000$$

Ex: For a host-parasite relation it was determined that when the host density is x (# of hosts per area) the number of host parasites is $y = \frac{900x}{10+45x}$, i.e. $\lim_{x \to \infty} \frac{900x}{10+45x} = \frac{900}{45} = 20$

If hosts increase without bound, what about parasites?

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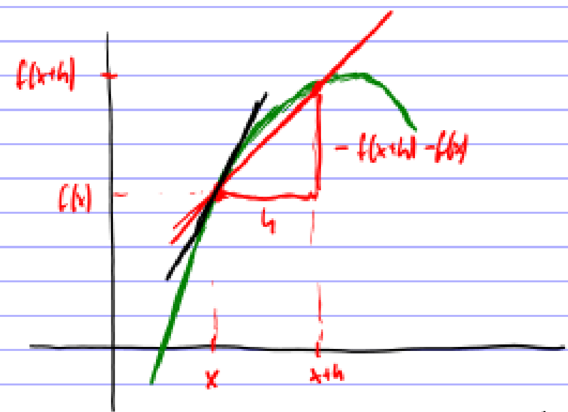
Panel 12

The Most Famous Limit of All:

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{f(x) - f(x)}{0} = \frac{0}{0}$$



$\frac{f(x+h) - f(x)}{h}$ is slope of line as shown

In the limit as $h \to 0$ line turns into tangent line (black)

Thus: $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ is slope of tangent line

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Panel 13

The Derivative

If $f(x)$ is a function, we define the derivative of f as

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided the limit exists.

Geometrically, $f'(x)$ is the slope of the tangent line.

Ex: Find $f'(x)$ if $f(x) = x^2$ (at $x = 1$, say)

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h} =$$

$$= \lim_{h\to 0} \frac{2xh + h^2}{h} = \lim_{h \to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}} = 2x$$

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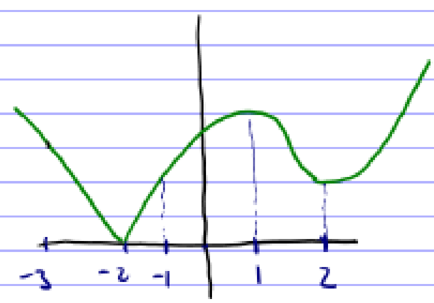
Panel 14

Ex: Find the indicated ~~quantities~~: slope of tangent



$f'(-3)$

$f'(-1)$

$f'(0)$

$f'(1)$

$f'(2)$

$f'(1.5)$

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