Panel 1

New Subject Entirely: Calculus (chapter 10 - 14)

Want to look at $f(x) = \frac{x^2-1}{x-1}$ Can't plug in $x=1$

Wonder what happens if I am close to 1

x		x	
0.8	1.8	1.2	2.2
0.9	1.9	1.1	2.1
0.99	1.99	1.01	2.01
0.999	1.999	1.001	2.001
⋮	↓ 2.0	⋮	↓ 2.0

1

Panel 2

Def of Limit: $\lim_{x \to a} f(x) = L$ (limit as x approaches a is L)

means that as x gets closer to a, without being equal to a, $f(x)$ gets closer to L

Ex: $\lim_{x \to 1} \frac{x^2-1}{x-1} = 2$

$\lim_{x \to 1} 2x+3 = 5$

$\lim_{x \to 0} \frac{x^2-4}{x-2} = 2$ BUT $\lim_{x \to 2} \frac{x^2-4}{x-2} \overset{\text{stuck}}{=} \lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = 4$

2

Panel 3

Limits may or may not exist!

$\lim_{x \to 0} \frac{1}{x^2} = \infty$ or error, does not exist!

Note: $\frac{0}{\#} = 0$ $\lim_{x \to 0} \frac{x}{x-1} = 0$

$\frac{\#}{0} =$ undefined $\lim_{x \to 1} \frac{x}{x-1} =$ undefined

$\frac{0}{0} =$ more work $\lim_{x \to 0} \frac{x}{x^2 - x} = \lim_{x \to 0} \frac{\cancel{x}\,1}{\cancel{x}(x-1)} = -1$

3

Panel 4

How to find limits:

$\lim_{x \to a} f(x)$: what happens to $f(x)$ as x gets closer and closer to a, but is not equal to a.

Step 1: plug in $x=a$ (illegal). If it works, that's the answer

Or: if $0/\# \Rightarrow 0$ or $\frac{\#}{0} \Rightarrow$ undefined

But: $0/0$ more work (usually factoring)

4

Panel 5

Find the following limits:

a) $\lim_{x\to 0} \frac{x-1}{x}$ undefined

b) $\lim_{x\to 1} \frac{x-1}{x} = 0$

c) $\lim_{x\to 0} \frac{x-1}{x^2-1} = 1$

d) $\lim_{x\to 1} \frac{x-1}{x^2-1} = \lim_{x\to 1} \frac{\cancel{x-1}}{(\cancel{x-1})(x+1)} = \frac{1}{2}$

5

Panel 6

Today: $\lim_{x\to 0} \frac{e^{2x}-1}{x} = 2$ (guess)

try x very small, say 0.0001

$\lim_{x\to 2} \frac{x^2-3x+2}{x^2-4} = \lim_{x\to 2} \frac{(\cancel{x-2})(x-1)}{(x+2)(\cancel{x-2})} = \lim_{x\to 2} \frac{x-1}{x+2} = \frac{1}{4}$

6

Panel 7

One - Sided Limit

Say we have $f(x) = \begin{cases} 3x^2 & \text{if } x<0 \\ 2x & \text{if } x \geq 0 \end{cases}$ Then, clearly

$\lim_{x \to 2} f(x) = 4$ and $\lim_{x \to -2} f(x) = 12$

What about $\lim_{x \to 0} f(x)$ - not sure

Def: $\lim_{x \to a^+} f(x)$ is x is close to a, but bigger than a

$\lim_{x \to a^-} f(x)$ is x is close to a, but less than a

7

Panel 8

Theorem: $\lim_{x \to a} f(x) = L$ if and only if

$\lim_{x \to 0^+} f(x) = L$ and $\lim_{x \to 0^-} f(x) = L$

Ex: $f(x) = \begin{cases} 3x^2 & \text{if } x<0 \\ 2x+1 & \text{if } x \geq 0 \end{cases}$ Find $\lim_{x \to 0} f(x)$

$$\left.\begin{array}{l} \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} 2x+1 = 1 \\ \\ \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} 3x^2 = 0 \end{array}\right\} \lim_{x \to 0} f(x) \text{ undefined}$$

8

Panel 9

$$f(x) = \begin{cases} 3x^2 - 1 & \text{if } x > 1 \\ 5x - 3 & \text{if } x \le 1 \end{cases}$$

$$\lim_{x \to 1} f(x) = ?\ 2$$

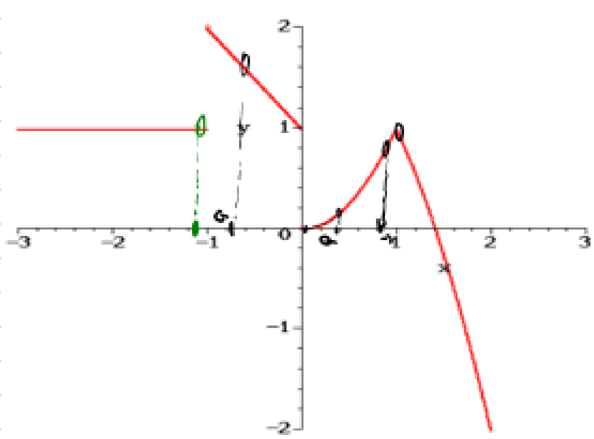
$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 3x^2 - 1 = 2$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} 5x - 3 = 2$$

9

Panel 10

Graphical limits



$$\lim_{x \to -1^-} f(x) = 1$$

$$\lim_{x \to -1^+} f(x) = 2$$

$$\lim_{x \to 0^-} f(x) = 1$$

$$\lim_{x \to 0^+} f(x) = 0$$

$$\lim_{x \to 1^-} f(x) = 1$$

$$\lim_{x \to 1^+} f(x) = 1$$

- pick x close to a on the correct side
- slide x towards a and watch f(x)

10

Panel 11

Limits at Infinity

Def: $\lim_{x \to \infty} f(x)$ is that # that $f(x)$ approaches as x gets larger + larger + larger ...

Ex: $\lim_{x \to \infty} \frac{1}{x^2} = 0$

$\lim_{x \to -\infty} x^3 = -\infty$ or undefined as a number

$\lim_{x \to -\infty} \frac{1}{x} = -0 = 0$

11

Panel 12

Limits at Infinity for Rational Functions

$$\lim_{x \to \infty} \frac{x^3 - 2x - 7}{3x^3 + 2} = \lim_{x \to \infty} \frac{\cancel{x^3}\left(1 - \frac{2}{x^2}^{\to 0} - \frac{7}{x^3}^{\to 0}\right)}{\cancel{x^3}\left(3 + \cancel{\frac{2}{x^3}}\right)} = \frac{1}{3}$$

$$\lim_{x \to \infty} \frac{x^2 - 7 - x}{x^3 - 7x + 9} = 0$$

$$\lim_{x \to \infty} \frac{x^4 - 7}{x^3 - 8x^2 + 9x} = \text{undefined}$$

Rule: think of it as a race and consider highest powers only.

12

Panel 13

Limits at Infinity:

$$\lim_{x \to \pm\infty} \frac{p(x)}{q(x)} = \begin{cases} \deg(\text{top}) < \deg(\text{bottom}) & : 0 \\ \deg(\text{top}) = \deg(\text{bottom}) & : \#/\# \\ \deg(\text{top}) > \deg(\text{bottom}) & : \text{undef } (\pm\infty) \end{cases}$$

Ex: a) $\lim_{x \to -\infty} \frac{x^2 - 7x + 9}{3 - 4x^2} = -1/4$ quiz on Wed

b) $\lim_{x \to \infty} \frac{3x^2 - 7}{9x^3 - 5} = 0$

c) $\lim_{x \to -\infty} \frac{3 - 4x + 5x^3}{7x^2 - 9} =$ undef $(= -\infty)$

13