

Panel 1

Last Time

Exp function: $y = b^x, b > 0$ or $y = e^x, e = 2.71828$.

Log function: $y = \log_b(x), y = \log(x), y = \ln(x)$
'10' 'e'

Properties of log-functions.

$$y^p = x \quad y^2 = x$$

$$\log_b(x) = y \Leftrightarrow b^y = x$$

$$\log_b(1) = 0 \quad \text{und} \quad \log_b(b) = 1$$

$$x = x \cdot \log_b(b) = \log_b(b^x) = x \quad \text{und} \quad b^{\log_b(x)} = x$$

$$\log_b(x^p) = p \log_b(x)$$

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Panel 2

Graphs of exp and log functions

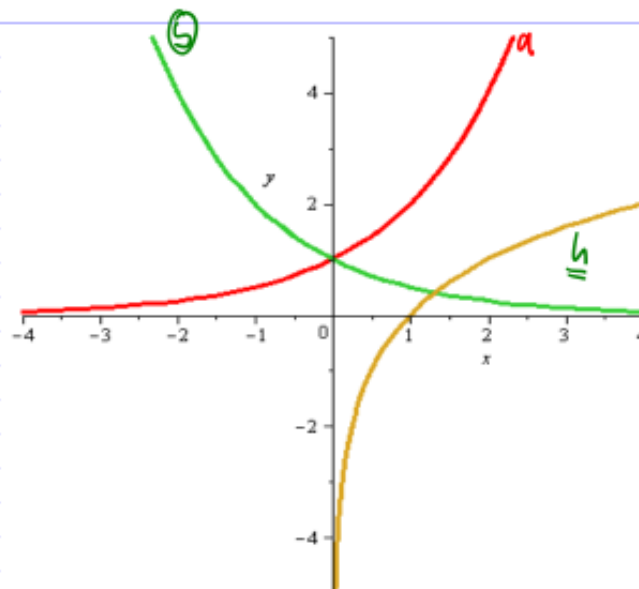
Identity

a) $f(x) = 2^x$

b) $g(x) = \left(\frac{1}{2}\right)^x$

c) $h(x) = \log_2(x)$

~~$\log_{1/2}(x)$~~



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Panel 3

Example: Solve $5 + 3 \cdot 4^{x-1} = 12$ isolate exp-funct
and log-it

$$3 \cdot 4^{x-1} = 7$$

$$4^{x-1} = \frac{7}{3} \quad | \log_4$$

$$\log_4(4^{x-1}) = \log_4\left(\frac{7}{3}\right)$$

$$x-1 = \log_4\left(\frac{7}{3}\right)$$

$$x = \log_4\left(\frac{7}{3}\right) + 1 = ?$$

$$4^{x-1} = \frac{7}{3} \quad | \log(\)$$

$$\log(4^{x-1}) = \log\left(\frac{7}{3}\right)$$

$$(x-1) \cdot \log(4) = \log\left(\frac{7}{3}\right)$$

$$x-1 = \frac{\log\left(\frac{7}{3}\right)}{\log(4)} \Rightarrow x = \frac{\log\left(\frac{7}{3}\right)}{\log(4)} + 1 = \frac{0.3619}{0.6020} + 1 = \dots$$

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Panel 4

Solve the equations $2^x = 100$ by

a) using \log_2 $\log_2(2^x) = \log_2(100)$
 $x = \log_2(100)$

b) using \log $\log(2^x) = \log(100)$
 $x \log(2) = 2 \Rightarrow x = \frac{2}{\log(2)} = \underline{6.6439}$

c) using $\ln(x)$ $\ln(2^x) = \ln(100)$
 $x \ln(2) = \ln(100)$
 $x = \frac{\ln(100)}{\ln(2)} = \underline{6.6439}$

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Panel 5

If you invest \$100 at 10% compounded monthly, when would you have \$1000?

$$S = P(1 + r)^n = 100 \left(1 + \frac{0.1}{12}\right)^{12 \cdot t} = 1000$$

$$\left(1 + \frac{0.1}{12}\right)^{12 \cdot t} = 10 \quad |$$

$$1.00833^{12t} = 10 \quad | \log$$

$$12 \cdot t \log(1.00833) = 1$$

$$t = \frac{1}{12 \cdot \log(1.00833)} = \underline{\underline{23.1}}$$

$$\approx 23 \text{ years}$$

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Panel 6

Suppose the number of milligrams of a radioactive substance is $N(t) = 100 e^{-0.05t}$. How long does it take for $\frac{1}{2}$ of the substance to disappear?

Note: Half-life is time it takes for $\frac{1}{2}$ substance to disappear.

$$N(0) = 100 \quad \text{want: } N(t) = 100 e^{-0.05t} = 50$$

$$e^{-0.05t} = \frac{50}{100} = \frac{1}{2} \quad | \ln()$$

$$-0.05t = \ln\left(\frac{1}{2}\right)$$

$$t = \frac{\ln\left(\frac{1}{2}\right)}{-0.05} = \underline{\underline{13.86}}$$

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Panel 7

Exam #1

(Review)

$$\frac{1}{\sqrt{x^2+1}}$$

want $x^2+1 > 0$ all \mathbb{R} 's

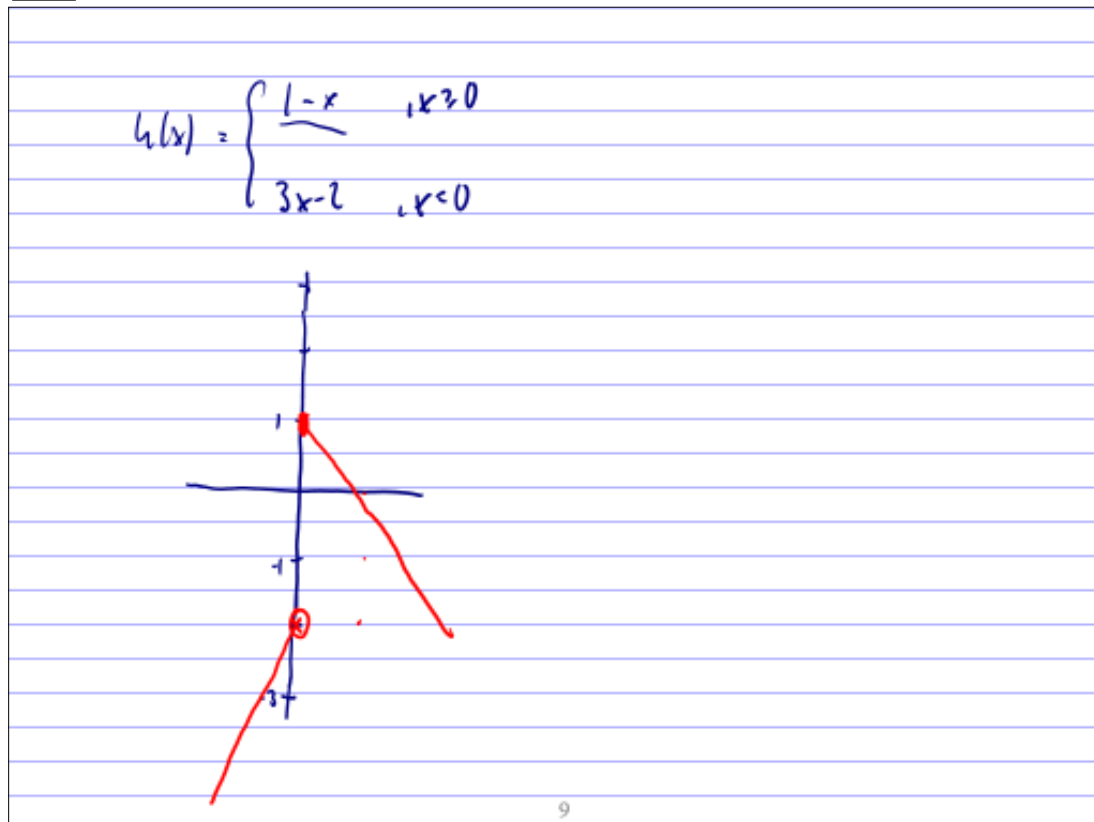
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Panel 8

$$\begin{aligned}
 f(x) = 2x^2 - 3 \quad \text{find } \frac{f(x+h) - f(x)}{h} &= \frac{(2(x+h)^2 - 3) - (2x^2 - 3)}{h} \\
 &= \frac{2(x^2 + 2xh + h^2) - 3 - 2x^2 + 3}{h} \\
 &= \frac{2x^2 + 4xh + 2h^2 - 2x^2}{h} \\
 &= \frac{4xh + 2h^2}{h} = h \frac{(4x + 2h)}{h} = \underline{\underline{4x + 2h}}
 \end{aligned}$$

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Panel 9



Panel 10

$$\begin{aligned} 5p - q &= 10 & \Rightarrow 5p - 10 &= q \\ 2p^2 - q &= 8 & 2p^2 - 8 &= q \end{aligned} \quad \Rightarrow \quad \frac{5p - 10 = 2p^2 - 8}{0 = 2p^2 - 5p + 2}$$

$$p = \frac{5 \pm \sqrt{25 - 16}}{4} = \frac{5 \pm \sqrt{9}}{4} = \begin{cases} \frac{5+3}{4} = 2 \\ \frac{5-3}{4} = \frac{1}{2} \end{cases}$$

$$\underline{4^{3-x} = \frac{1}{16}} \quad | \log_4$$

$$\log_4(4^{3-x}) = \log_4\left(\frac{1}{16}\right) = \log_4(4^{-2}) = -2 \log_4(4) = -2$$

$$3-x = -2 \quad \underline{x = 5}$$