Panel 1

Last Time

Exp function: $y = b^x,\ b>0$, $y = e^x$, $e = 2.7182\ldots$

Log function: $y = \log_b(x)$, $y = \log(x)$ (base 10), $y = \ln(x)$ (base e)

Properties of log-functions.

$\sqrt{x} = y \iff y^2 = x$

!! $\log_b(x) = y \iff b^y = x$

! $\log_b(1) = 0$ and $\log_b(b) = 1$

$x \log_b(b) = \log_b(b^x) = x$ and $b^{\log_b(x)} = x$

! $\log_b(x^p) = p \cdot \log_b(x)$

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Panel 2

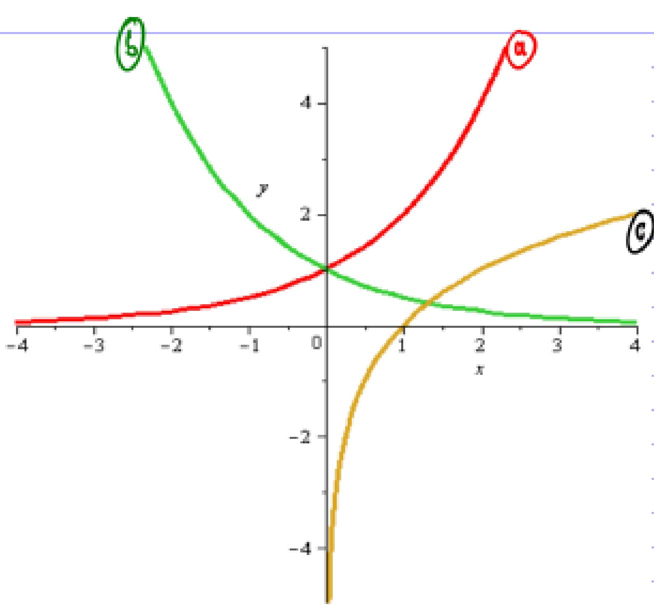
Graphs of exp and log functions

Identify

a) $f(x) = 2^x$

b) $g(x) = \left(\frac{1}{2}\right)^x$

c) $h(x) = \log_2(x)$



$\log_{\cancel{2}}(x)$

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Panel 3

isolate exp. function

Example: Solve $5 + 3 \cdot 4^{x-1} = 12$

$$3 \cdot 4^{x-1} = 7$$

$$4^{x-1} = 7/3 \quad | \log_4(\ )$$

$$\cancel{\log_4}(\cancel{4}^{x-1}) = \log_4(7/3)$$

$$x-1 = \log_4(7/3) \quad \Rightarrow \quad \underline{\underline{x = \log_4(7/3)+1}}$$

Alternative: $4^{x-1} = 7/3 \quad | \log(\ )$

$$\log(4^{x-1}) = \log(7/3)$$

$$(x-1)\log(4) = \log(7/3) \quad \Rightarrow \quad x-1 = \frac{\log(7/3)}{\log(4)} \quad \Rightarrow \quad x = \frac{\log(7/3)}{\log(4)} + 1$$

$$x = \frac{0.3679}{0.602} + 1 = \dots$$

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Panel 4

Solve the equation $2^x = 100$ by

a) using $\log_2$: $\log_2(2^x) = \log_2(100)$

$$\underline{\underline{x = \log_2(100)}}$$

b) using $\log$: $\log(2^x) = \log(100)$

$$x \cdot \log(2) = 2 \quad \Rightarrow \quad x = 2/\log(2) = \underline{6.6439}$$

c) using $\ln(x)$ $\ln(2^x) = \ln(100)$

$$x \ln(2) = \ln(100)$$

$$x = \frac{\ln(100)}{\ln(2)} = \underline{\underline{6.6439}}$$

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Panel 5

If you invest \$100 at 10% compounded monthly, when would you have \$1000?

$$S = P(1+r)^n = \underbrace{100\left(1+\frac{0.1}{12}\right)^{12\cdot t} = 1000}$$

$$(1.00833)^{12t} = 10 \quad | \log$$

$$\log\left[(1.00833..)^{12t}\right] = \log(10)$$

$$12t \log(1.00833..) = 1$$

$$t = \frac{1}{12\cdot\log(1.00833)} = 23 \text{ years}$$

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Panel 6

Suppose the number of milligrams of a radioactive substance is $N(t) = \underline{100e^{-0.05t}}$. How long does it take for ½ of the substance to disappear?

Half-Life is time it takes for ½ substance to disappear.

Start: $N(0) = 100$

want: $\underline{N(t)} = \underline{50 = 100e^{-0.05\cdot t}}$

$$\frac{1}{2} = e^{-0.05\cdot t} \quad | \ln$$

$$\ln(1/2) = \cancel{\ln}(\cancel{e}^{-0.05\cdot t}) = -0.05t$$

$$\text{years } \underline{13.86} = \frac{\ln(1/2)}{-0.05} = t$$

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Panel 7

Exam #1

(Review)

$f(x) = 2x^2 - 3$

$$\frac{f(x+h) - f(x)}{h} = \frac{(2(x+h)^2 - 3) - (2x^2 - 3)}{h}$$

$$= \frac{2(x^2 + 2xh + h^2) - \cancel{3} - 2x^2 + \cancel{3}}{h}$$

$$= \frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{2x^2}}{h}$$

$$= \frac{h(4x + 2h)}{h} = 4x + 2h$$

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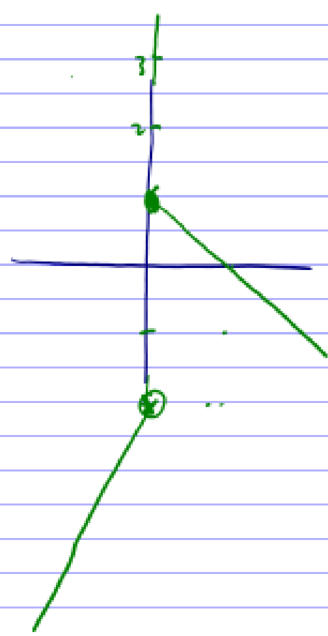
Panel 8

$$h(x) = \begin{cases} 1 - x & \text{if } x \geq 0 \\ 3x - 2 & \text{if } x < 0 \end{cases}$$



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Panel 9

$$5p - q = 10 \quad \Rightarrow \quad 5p - 10 = q$$

$$2p^2 - q = 8 \qquad\quad 2p^2 - 8 = q$$

$$\Rightarrow \underline{5}p - \underline{10} = 2p^2 - 8$$

$$\underline{\underline{0 = 2p^2 - 5p + 2}}$$

$$p = \frac{5 \pm \sqrt{25 - 16}}{4} =$$

$$= \frac{5 \pm \sqrt{9}}{4} = \frac{5 \pm 3}{4} = \begin{cases} 8/4 = \underline{\underline{2}} \\ 2/4 = \underline{\underline{1/2}} \end{cases}$$

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