Panel 1

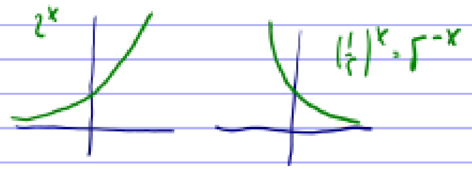
Last Time

$f(x) = b^x, \ b > 0$

Exp. function, Properties.

domain: $\mathbb{R}$

range: $(0, \infty)$



Compound Interest: $S = P(1+r)^n$

P = principal, r = rate per period, n = # of periods

1

Panel 2

2000 invest at 9.9% cpd. annually. When does it at least double?

monthly

for 8 years

Later

$$S = 2000\left(1 + \frac{0.09}{12}\right)^{8 \cdot 12}$$

$$\frac{350000}{(1.015)}$$

$$P = P(1-r)^t$$

2

Panel 3

Name:

Quiz #3

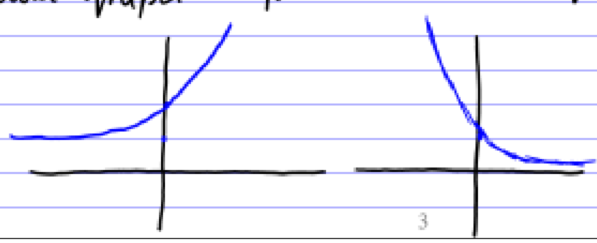
① The amount of a radioactive substance in gr is:

$$M(t) = 128\left(\frac{3}{4}\right)^t$$

a) How much material is present initially

b) How much is present after 3 days

② Which graph represents which function:



a) $f(x) = 2^x + 1$

b) $g(x) = \left(\frac{1}{3}\right)^x$

3

Panel 4

③ Suppose a principle $P = \$2000$ is invested over 5 years at an annual rate of 4%

a) What is the amount if compounded annually?

b) What is the amount for weekly compounding?

4

Panel 5

Name:

Quiz #3

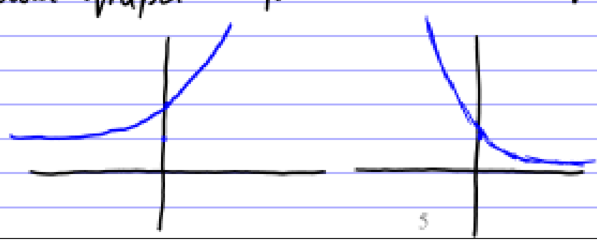
1. The amount of a radioactive substance in gr is:

$$M(t) = 54\left(\frac{2}{3}\right)^t$$

a) How much material is present initially

b) How much is present after 3 days

2. Which graph represents which function:



a) $f(x) = \left(\frac{1}{3}\right)^x$

b) $g(x) = 2^x + 1$

5

Panel 6

3. Suppose a principle $P = \$2000$ is invested over 4 years at an annual rate of 5%

a) What is the amount if compounded annually?

b) What is the amount for weekly compounding?

3.298 E03

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Panel 7

Thought Experiment: Invest \$1 at 100% for one year, compounded n-times per year:

$$S = 1 \cdot \left(1 + \frac{1}{n}\right)^n = \left(1 + \frac{1}{n}\right)^n$$

n	S
1	2
10	$\left(1+\frac{1}{10}\right)^{10} = 2.59$
100	2.7048
1000	2.71692
10000	2.71815...
100000	2.7182...

$\leftarrow e$

$e = \left(1+\frac{1}{n}\right)^n$, n very large

Euler's # 2.7182...

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Panel 8

The Natural Exponential Function

Define the number $e = 2.7182\ldots$ (Euler's Number)

Then $f(x) = e^x$ is natural exp. function

Ex: $f(0) = 1$

$f(3) = 20.085\ldots$

$f(-2) = 0.1353\ldots$

$f(1/3) = 1.396\ldots$

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Panel 9

Population Growth: The projected population P of a city is

$$P(t) = 100\,000\, e^{\underbrace{0.05t}_{\text{positive} \to \text{increasing}}}$$

where t is the number of years after 1990.

a) What was the population in 1990

$P(0) = 100\,000$

b) Predict population in 2020

$P(30) > 100\,000$

9

Panel 10

Radioactive Decay: A radioactive element decays such that after t days the amount left (in mg) is

$$N = 100\, e^{\underbrace{-0.062t}_{\text{negative} \to \text{decreasing}}}, \quad t \text{ in days}$$

a) How many mg are initially present?

$N(0) = 100$

b) How much after 10 days?

$2^{-x} = \frac{1}{2^x} = \left(\frac{1}{2}\right)^x$

$N(10) < 100$

10

Panel 11

Logarithm Functions:

$y = x^2 \iff \sqrt{y} = x$, i.e. $\sqrt{y}$ is that number x s.t. $x^2 = y$

e.g. $\sqrt{9} = 3$

Definition $\log_b(x) = y \iff b^y = x$

base b

Note: Just as $\sqrt{x^2} = x = (\sqrt{x})^2$

$b^{\log_b(x)} = x = \log_b(b^x)$

Technically: inverse functions

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Panel 12

Some examples:

Find the following values:

$\log_5(25) = y = 2$	$5^y = 25$	Fact:
$\log_3(81) = y = 4$	$3^y = 81$	$\log_5(1) = 0$
$\log_{10}(1) = y = 0$	$10^y = 1$	$\log_b(b) = 1$
$\log_2(\frac{1}{16}) = y = -4$	$2^y = 1/16$	
$\log_{64}(8) = y = 1/2$	$64^y = 8$	
$\log_{10}(10) = y = 1$	$10^y = 10$	

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Panel 13

Graph $y = \log_2(x)$ and $y = 2^x$ in one system

x	y
1	0
2	1
3	
4	2
8	3
16	4
~~0~~	
~~x~~	
$\frac{1}{2}$	-1
$\frac{1}{4}$	
$\frac{1}{8}$	
$\frac{1}{16}$	

$\log_2(1) = 0$

$\log_2(2) = 1$

$\log_2(3) = y$

$2^y = 3$

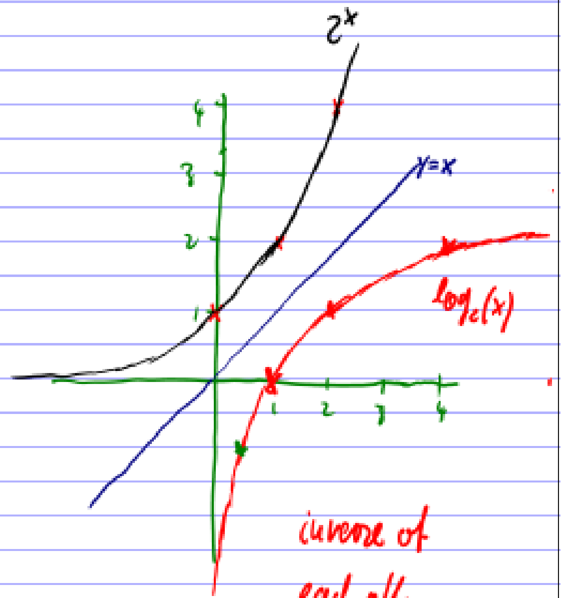
$\log_2(4) = 2$

$\log_2(0) = y$

$2^y = 0$

$\log_2(-1) = y$

$2^y = -1$



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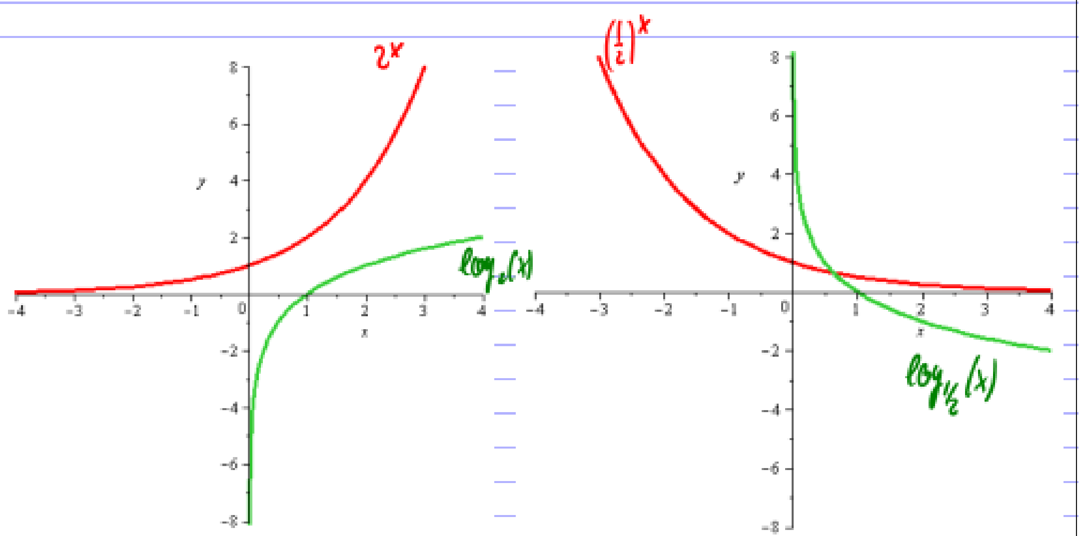
Panel 14

Graphs of 5^x and $\log_5$



Domain: $(0, \infty)$

Range: $\mathbb{R}$

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Panel 15

Solve the following equations:

a) $\log_2(x) = 4$ $\qquad 2^4 = x$

$x = 16$

b) $\log_3(x+1) = 7$ $\qquad 3^7 = x+1, \ x = 3^7 - 1$

c) $\log_x(49) = 2$ $\qquad x^2 = 49, \ x = 7$

d) $2^{5x} = 4 = 2^2$ $\qquad \Rightarrow 5x = 2, \ x = \frac{2}{5}$

$2^{5 \cdot \frac{2}{5}} = 2^2 = 4$ ✓

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Panel 16

Special Logarithms

$\log_b(x) = y \iff$

Natural logarithm: $\log_e(x) = \ln(x)$ (Base e)

Common logarithm: $\log_{10}(x) = \log(x)$ (Base 10)

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Panel 17

Properties of Logarithms

$\log_5(1) = 0$

$\log_5(5) = 1$

domain of $\log_5(x)$: $(0, \infty)$

range of $\log_5(x)$: $\mathbb{R}$

$\log_5(5^x) = x$

$5^{\log_5(x)} = x$

$\log_5(x^p) = p \log_5(x)$ why log is useful!

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Panel 18

Examples

Solve $25^{x+2} = 5^{3x-4}$

$(5^2)^{x+2} = 5^{3x-4}$

$5^{2x+4} = 5^{3x-4} \Rightarrow 2x+4 = 3x-4 \Rightarrow x = 8$

Solve $5^{2x-1} = 125$ and $5^{2x-1} = 100$ $\quad |\log_5(\ )$

easy

$\log_5(5^{2x-1}) = \log_5(100)$

$2x-1 = \log_5(100)$

$x = \frac{1}{2}(\log_5(100)+1)$

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Panel 19

$5^{2x-1} = 100 \quad | \log \quad (\log_{10})$

$\log(5^{2x-1}) = \log(100)$

$(2x-1)\log(5) = 2$

$2x-1 = \frac{2}{\log(5)} = 2.861$

...

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Panel 20

Q. Invest 2000 at 9.9% comp. annually. When does it double

When does it reach $22 538

$2000(1+0.099)^t = 4000$

$(1.099)^t = 2 \quad | \log$

$\log(1.099^t) = \log(2)$

$t \cdot \log(1.099) = \log(2)$

$t = \frac{\log(2)}{\log(1.099)} = \frac{0.3010}{0.04099} = 7.7418$

or 8 years

20

Panel 21

Ex on Wed.

Wrap up + review on Monday

Practice exam posted over weekend

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