Panel 1

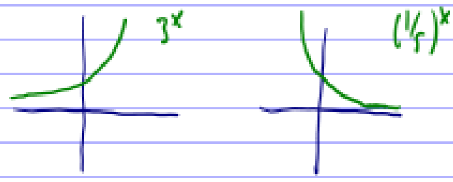
Last Time

Exp. function, Properties.

$f(x) = b^x$, $b > 0$

domain: $\mathbb{R}$

range: $(0, \infty)$



Compound Interest: $S = P(1+r)^n$

P principal, r = rate per period, n = # of periods

1

Panel 2

Quiz #3 Name:

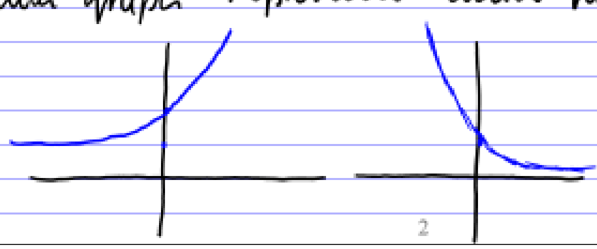
① The amount of a radioactive substance in gr is:

$$M(t) = 128\left(\frac{3}{4}\right)^t$$

a) How much material is present initially

b) How much is present after 3 days

② Which graph represents which function:



a) $f(x) = 2^x + 1$

b) $g(x) = \left(\frac{1}{3}\right)^x$

2

Panel 3

(3) Suppose a principle $P = \$2000$ is invested over 5 years at an annual rate of 4%

a) What is the amount if compounded annually?

b) What is the amount for weekly compounding?

3

Panel 4

Name:

Quiz #3

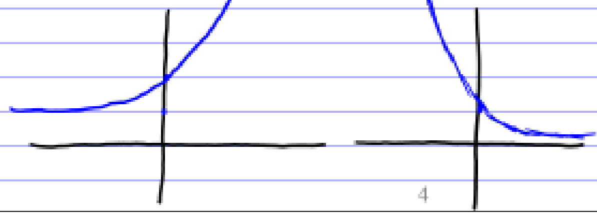
(1) The amount of a radioactive substance in gr is:

$$M(t) = 54\left(\frac{2}{3}\right)^t$$

a) How much material is present initially

b) How much is present after 3 days

(2) Which graph represents which function:



a) $f(x) = \left(\frac{1}{3}\right)^x$

b) $g(x) = 2^x + 1$

4

Panel 5

(3) Suppose a principle $P = \$2000$ is invested over 4 years at an annual rate of 5%

a) What is the amount if compounded annually?

b) What is the amount for weekly compounding?

5

Panel 6

Thought Experiment: Invest \$1 at 100% for one year, compounded n-times per year:

$$S = 1 \cdot \left(1 + \frac{1}{n}\right)^n = \left(1 + \frac{1}{n}\right)^n$$

n	S
1	2
10	$\left(1+\frac{1}{10}\right)^{10} = 2.59$
100	2.705
1000	2.71692
10000	2.71815
100000	2.71827
	⋮

$$e = \left(1 + \frac{1}{n}\right)^n, \text{ n large.}$$

6

Panel 7

The Natural Exponential Function

Define the number $e = 2.7182\ldots$ (Euler's Number)

Then $f(x) = e^x$ is natural exp. function

Ex: $f(0) = 1$

$f(3) = 20.085$

$f(-2) = 0.135\ldots$

$f(1/3) = 1.3956\ldots$

7

Panel 8

Population Growth: The projected population P of a city is $P(t) = 100\,000\, e^{0.05t}$ pos. ⇒ grows

where t is the number of years after 1990.

a) What was the population in 1990

$P(0) = 100{,}000$

b) Predict population in 2020

$P(30) = \ldots > 100{,}000$

8

Panel 9

Radioactive Decay: A radioactive element decays such that after t days the amount left (in mg) is

$$N = 100 e^{\underbrace{-0.062}_{\text{negative = decay}} t}, \quad t \text{ in days}$$

a) How many mg are initially present?

$N(0) = 100$

b) How much after 10 days?

$2^{-x} = \frac{1}{2^x} = \left(\frac{1}{2}\right)^x$

$\underline{N(10) < 100}$

9

Panel 10

Logarithm Functions:

Recall: $y = x^2 \Leftrightarrow \sqrt{y} = x$, i.e. $\sqrt{y}$ is that # x s.t. $x^2 = y$, e.g. $\sqrt{9} = 3$

Define: $y = \log_5(x) \Leftrightarrow 5^y = x$ (base 5)

Thus: just as $\sqrt{x^2} = (\sqrt{x})^2$ we have

$\cancel{\log_5}(\cancel{5}^x) = x$, $\cancel{5}^{\cancel{\log_5}(x)} = x$

10

Panel 11

Some examples:

$y = \log_5(x) \Leftrightarrow 5^y = x$

Find the following values:

$y = \log_5(25) = \underline{2} \Leftrightarrow 5^y = 25$

$y = \log_3(81) = \underline{4} \Leftrightarrow 3^y = 81$

$y = \log_{10}(1) = \underline{0} \Leftrightarrow 10^y = 1$

$y = \log_2\left(\frac{1}{16}\right) = \underline{-4} \qquad 2^y = \frac{1}{16} = 2^{-4}$

$y = \log_{64}(8) = \underline{1/2} \qquad 64^y = 8$

$y = \log_{17}(17) = 1 \qquad 17^y = 17$

Fact

$\log_6(1) = 0$

$\log_6(6) = 1$

11

Panel 12

Graph $y = \log_2(x)$ and $y = 2^x$ in one system

x	y
1	0
~~0~~	
2	1
3	?
4	2
8	3
16	4
32	5
64	6

$y = \log_2(0)$
$2^y = 0$

$y = \log_2(3)$
$2^y = 3$

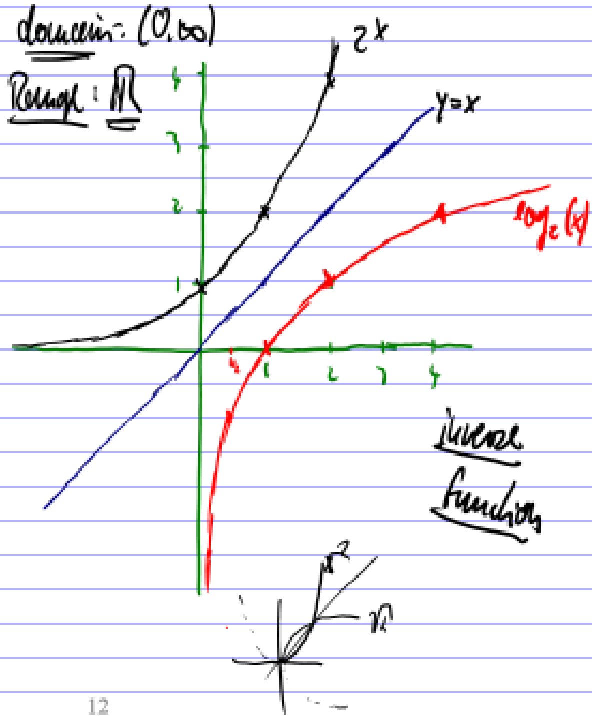
$y = \log_2(4)$
$2^y = 4$

$y = \log_2(\cancel{x})$
$2^y = -1$

$y = \log_2\left(\frac{1}{2}\right)$

domain: $(0, \infty)$

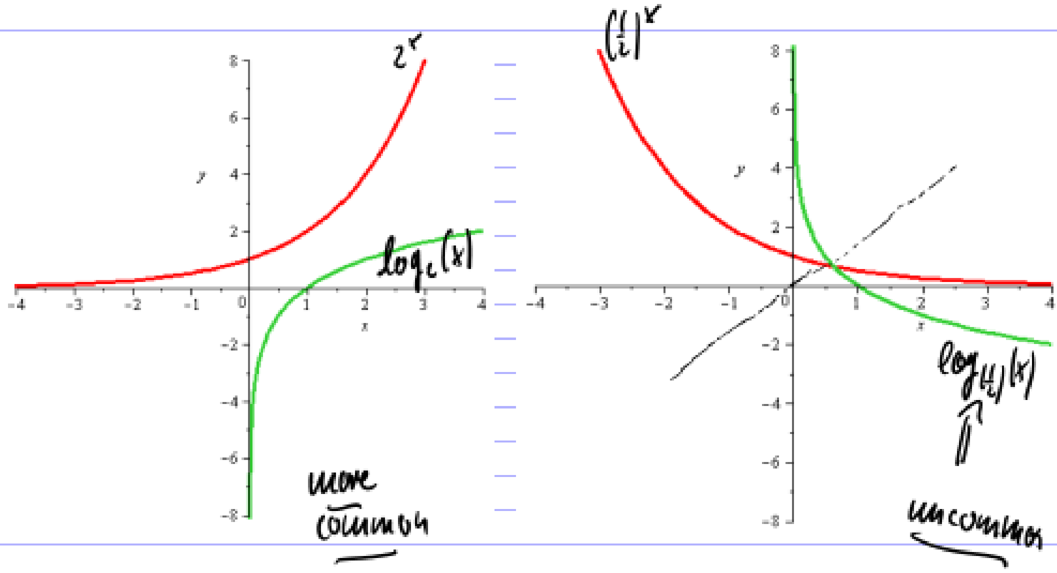
Range: $\mathbb{R}$



12

Panel 13

Graphs of b^x and $\log_b$



13

Panel 14

Solve the following equations:

a) $\log_2(x) = 4$ $\qquad 2^4 = x = 16$

b) $\log_3(x+1) = 7$ $\qquad 3^7 = x+1 \Rightarrow x = 3^7 - 1$

c) $\log_x(49) = 2$ $\qquad x^2 = 49 \Rightarrow x = 7$

d) $2^{5x} = 4$ $\qquad 2^{5x} = 2^2 \Rightarrow 5x = 2,\ x = 2/5$

14

Panel 15

Special Logarithms

$\log_b(x) = y \iff b^y = x$

Natural logarithm: $\log_e(x) = \ln(x)$ (base e)

Common logarithm: $\log_{10}(x) = \log(x)$ (base 10)

$\log(1000) = \cancel{y}\ 3,\ 10^y = 1000$

15

Panel 16

Properties of Logarithms

$\log_b(1) = 0$

$\log_b(b) = 1$

$\log_b(x^p) = p \log_b(x)$

$\log_b(b^x) = x = b^{\log_b(x)}$

$\left(\log_b(x) = y \iff b^y = x\right)$

16

Panel 17

Examples

Solve $25^{x+2} = 5^{3x-4}$

$(5^2)^{x+2} = 5^{3x-4}$

$5^{2(x+2)} = 5^{3x-4} \Rightarrow 2x+4 = 3x-4$

$8 = x$

Solve $5^{2x-1} = 125 \Rightarrow 5^{2x-1} = 5^3 \Rightarrow 2x-1 = 3$

But: $5^{2x-1} = 100 \quad | \log_5(\,)$

no $\log_5$ button on calc

$\log_5(5^{2x-1}) = \log_5(100)$

$2x-1 = \log_5(100) \Rightarrow 2x = \log_5(100)+1 \quad x = \dfrac{\log_5(100)+1}{2}$

17

Panel 18

Examples

Simplify $e^{x \log_e(5)}$

18

Panel 19

$5^{2x-1} = 100 \quad | \log$ (because my calc. has LOG button)

$\log(5^{2x-1}) = \log(100)$

$(2x-1)(\log(5)) = \log(100)$

$2x-1 = \frac{\log(100)}{\log(5)}$

$2x = \frac{\log(100)}{\log(5)} + 1$

$x = \frac{1}{2}\left(\frac{\log(100)}{\log(5)} + 1\right)$

19

Panel 20

Finish up Monday

Practice Exam posted over weekend

Review on Mon

Ex 1 on Wed in class

20