Panel 1

Last Time

Quadratic functions: $f(x) = ax^2 + bx + c$ (a: down/up)

vertex: $x = -b/2a$, $y = f(-b/2a)$

x-int: $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Applications: $R = p \cdot q$

$C = c_{fc} + c_{vc}$

$P = R - C$

Break-Even $R = C$

Equilibrium demand = supply

Mixture problems

1

Panel 2

Not all systems of equations are linear:

Solve $x^2 - 2x + y - 7 = 0$

$3x - y + 1 = 0$

$x^2 + x - 6 = 0$ $(x=-3 \Rightarrow y=-8)$

$(x+3)(x-2) = 0$ $\Rightarrow (x=2 \Rightarrow y=7)$

Solve $y = \sqrt{x+2}$ check answers

$x + y = 4$

$x + \sqrt{x+2} = 4$

$(\sqrt{x+2})^2 = (4-x)^2 \quad | ( )^2$

$x+2 = 16 - 8x + x^2$

$0 = 14 - 9x + x^2$

$0 = x^2 - 9x + 14$

$0 = (x-2)(x-7)$

$\begin{cases} x=2 \\ y=2 \end{cases}$ $\begin{cases} \cancel{x=7} \\ \cancel{y=-3} \end{cases}$

2

Panel 3

A cost function is given as $C(q) = 2q^2 + 10$. Each item sells for \$20. What is the fixed cost? Find the profit function. What is the break-even point?

Fixed cost: $C(0) = 10$

~~Profit~~ Rev. $R(q) = 20q$

$$P(q) = 20q - (2q^2 + 10)$$

Break-Even: $20q = 2q^2 + 10$

$$0 = 2q^2 - 20q + 10 = 2(q^2 - 10q + 5)$$

$$q = \frac{10 \pm \sqrt{100 - 20}}{2} = \frac{10 \pm \sqrt{80}}{2} = \begin{cases} 9.47 \\ 0.523 \end{cases}$$

3

Panel 4

39.A company makes three types of patio furniture: chairs, rockers, and chaise lounges. Each requires wood, plastic, and aluminum, in the amounts shown in the following table. The company has in stock 400 units of wood, 600 units of plastic, and 1500 units of aluminum. For its end-of-the-season production run, the company wants to use up all the stock. To do this, how many chairs, rockers, and chaise lounges should it make?

	Wood	Plastic	Alminum
Chair	1 unit	1 unit	2 units
Rocker	1 unit	1 unit	3 units
Chaise lounge	1 unit	2 units	5 units

x = chairs

y = rockers

z = C.L.

Variables:

Wood used: $x + y + z = 400$

Plastic used: $x + y + 2z = 600$

Aluminum: $2x + 3y + 5z = 1500$

Input interpretation:

solve $x + y + z = 400$, $x + y + 2z = 600$, $2x + 3y + 5z = 1500$

Result:

$z = 200$ and $x = 100$ and $y = 100$

4

Panel 5

Chapter 3 - Review

Lines:

Applications: demand, supply

Quadratic functions: ✓

Systems of Equations: 2 ways to solve
non-linear systems

Applications: see previous slides.

5

Panel 6

Chapter 4: Exponential and Logarithm Functions

Recall: Rules of Exponents

$b^x = \underbrace{b \cdot b \cdot b \cdot \ldots \cdot b}_{x\text{-times}}$ $b^0 = 1$

$b^x \cdot b^y = b^{x+y}$ same base - powers add

$(b \cdot c)^x = b^x c^x$

$b^{-x} = 1/b^x$ negative exp. - flip

add

$(b^x)^y = b^{xy}$ powers to powers - multiply

$b^{1/2} = \sqrt{b}$, $b^{1/4} = \sqrt[4]{b}$

6

Panel 7

Example: Simplify the following expressions:

$$\frac{x^2 y^3}{x^4 y} = \frac{y^2}{x^2} = x^2 y^3 x^{-4} y^{-1} = x^{2-4} y^{3-1} = x^{-2} \cdot y^2 = \frac{y^2}{x^2}$$

$$\frac{x^2 + x^3 y^2}{\sqrt{x}\, y} = \frac{x^2(1+xy^2)}{x^{1/2} y} = x^2 \cdot x^{-1/2} \frac{1+xy^2}{y} = x^{3/2} \frac{1+xy^2}{y} = \sqrt{x^3}\, \frac{1+xy^2}{y}$$

$$\frac{x^2 y^{-3}}{\sqrt[4]{x^3}\, y^{-5}} = \frac{x^2 y^5}{y^3 x^{3/4}} = x^2 x^{-3/4} y^2 = x^{5/4} y^2$$

$$\frac{a^2 - b^2}{(a+b)^2 a^2} = \frac{(a+\cancel{b})(a-b)}{(a+b)^{\cancel{2}} a^2} = \frac{a-b}{(a+b)a^2}$$

7

Panel 8

Exponential and Logarithm Functions

Def: $f(x) = b^x$, $b > 0$, exponential function with base b.

Ex: Number of bacteria after t minutes is given by

$$N(t) = 300\left(\frac{4}{3}\right)^t$$

a) How many bacteria are present initially?

$$N(0) = 300\left(\frac{4}{3}\right)^0 = 300 \cdot 1$$

b) How many after 3 minutes?

$$N(3) = 300\left(\frac{4}{3}\right)^3 = 300\,\frac{64}{27} =$$

8

Panel 9

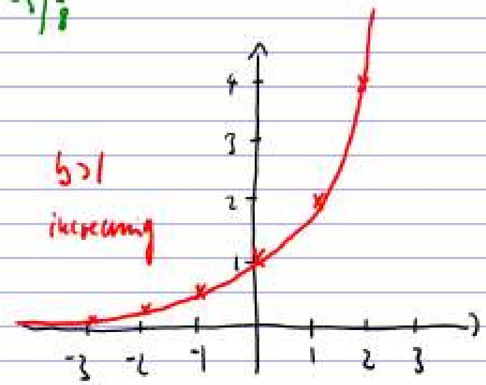
Graphs of Exponential Functions

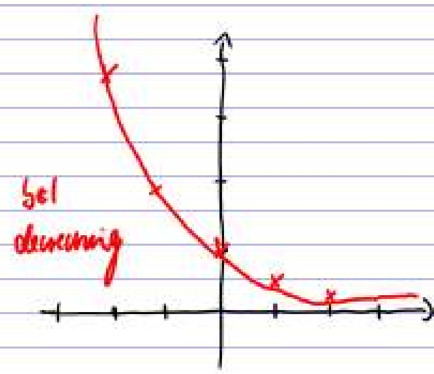
Graph $f(x) = 2^x$

x	y
2	4
1	2
0	1
-1	1/2
-2	1/4
-3	$\frac{1}{8}$

Graph $f(x) = \left(\frac{1}{2}\right)^x$

x	y
2	1/4
1	1/2
0	1
-1	2
-2	4





9

Panel 10

Properties of $f(x) = b^x$

Domain: $\mathbb{R}$

Range: $(0, \infty)$

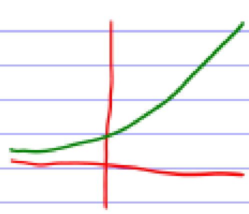
y-intercept: 1

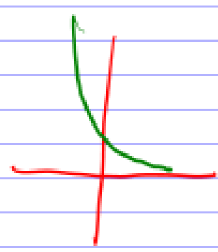
x-intercept: none

graph

inc. or decreasing

very fast





10

Panel 11

Exp. function is growing super-fast:

A sultan wanted to reward his wise man for his services and asked: "what do you want?" The wise man thought briefly, then said: "Put one grain of rice on a chess board field and double it for each subsequent field".

$1=2^0$	$2=2^1$	$4=2^2$
$8=2^3$	$16=2^4$	$32=2^5$
$64=2^6$	$128=2^7$	$256=2^8$

chessboard has 64 squares

$\Rightarrow 2^{63}$ grains of rice

9223372036854775808

9.223×10^{18} huge amount!

11

Panel 12

Application: Compound Interest

Suppose \$100 is invested at 5% interest, compounded annually. Find total amount after 2 years:

year 0: 100 year 1: $100 + 0.05 \cdot 100 = 105$, year 2: $105 + 0.05 \cdot 105 = 110.25$

In general: P principle, r = rate of interest

year 0: P

year 1: $P + r \cdot P = P(1+r)$

year 2: $P(1+r) + r \cdot P(1+r) = P(1+r)(1+r) = P(1+r)^2$

3: $P(1+r)^2 + r\,P(1+r)^2 = P(1+r)^2(1+r) = P(1+r)^3$

12

Panel 13

The compound amount S of a principle P at the end of n interest periods at a rate r per period is

$$S(n) = P(1+r)^n \quad \text{(exp. function)}$$

Ex: \$1000 invested over 10 years at 6% annually: no compounding: \$1600

$$S = 1000(1+0.06)^{10} = \$1790.85$$

\$1000 over 10 years at 6% annually, compounded monthly

$$S = 1000\left(1+\frac{0.06}{12}\right)^{120} = \$1819.40$$

\$1000 over 10 years at 6% annually, compounded daily:

$$S = 1000\left(1+\frac{0.06}{365}\right)^{3650} = \$1822.03$$

13

Panel 14

Thought Experiment: Invest \$1 at 100% for one year, compounded n-times per year:

later

14

Panel 15

The Natural Exponential Function

Define the number $e = 2.7182...$ (Euler's number)

$\pi = 3.1415...$

$f(x) = e^x$ - natural exp. function

$f(2) = 7.388$

$f(1/2) = \sqrt{2.7188}$

15