

Panel 1

Last Time

Quadratic functions:  $f(x) = ax^2 + bx + c$  (up/down)

vertex:  $x = -b/2a$

x-ints:  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Applications:

- Revenue  $R = pq$
- $C = C_{fc} + C_{vc}$
- $P = R - C$
- Break-Even point
- Equilibrium point (demand = supply)
- Mixing Problems

Panel 2

Not all systems of equations are linear:

Solve  $x^2 - 2x + y - 7 = 0$

$3x - y + 1 = 0$

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$x^2 + x - 6 = 0$

$(x+3) || (x-2) = 0$

$x = -3 \Rightarrow y = -8$

$x = 2 \Rightarrow y = 7$

Solve  $y = \sqrt{x+2}$

$x + y = 4$

*check answer*

$x + \sqrt{x+2} = 4$

$(\sqrt{x+2})^2 = (4-x)^2 \quad |(-1)^2$

$x+2 = 16 - 8x + x^2$

$0 = 14 - 9x + x^2$

$0 = x^2 - 9x + 14$

$0 = (x-7)(x-2)$

$x = 7, y = -3$  and  $x = 2, y = 2$

Panel 3

A cost function is given as  $C(q) = 2q^2 + 10$ . Each item sells for \$20. What is the fixed cost? Find the profit function. What is the break-even point?

$$\text{Fixed cost} = C(0) = 10$$

$$R(q) = 20q$$

$$P(q) = R(q) - C(q) = 20q - (2q^2 + 10)$$

$$R(q) = C(q) \quad (\Leftrightarrow) \quad 20q = 2q^2 + 10$$

$$0 = 2q^2 - 20q + 10 = 2(q^2 - 10q + 5)$$

$$q = \frac{+10 \pm \sqrt{100 - 20}}{2} = \frac{10 \pm \sqrt{80}}{2} = \begin{cases} \underline{9.47} \\ \underline{0.527} \end{cases}$$

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Panel 4

39. A company makes three types of patio furniture: chairs, rockers, and chaise lounges. Each requires wood, plastic, and aluminum, in the amounts shown in the following table. The company has in stock 400 units of wood, 600 units of plastic, and 1500 units of aluminum. For its end-of-the-season production run, the company wants to use up all the stock. To do this, how many chairs, rockers, and chaise lounges should it make?

	Wood	Plastic	Aluminum
Chair	1 unit	1 unit	2 units
Rocker	1 unit	1 unit	3 units
Chaise lounge	1 unit	2 units	5 units

 $x = \text{chairs}$ 
 $y = \text{rockers}$ 
 $z = \text{c.l.}$ 

Variables:

Wood used:  $x + y + z = 400$

Plastic used:  $x + y + 2z = 600$

Aluminum:  $2x + 3y + 5z = 1500$

	$x + y + z = 400$
solve	$x + y + 2z = 600$
	$2x + 3y + 5z = 1500$

Result:

$$z = 200 \text{ and } x = 100 \text{ and } y = 100$$

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Panel 5

## Chapter 3 - Review

Lines: slope, y-int., point-slope formula, general  
parallel, perpendicular

Applications: demand / supply - equilibrium point

Quadratic functions: vertex, intercepts, quadratic formula

Systems of Equations: 2 ways to solve

Applications: break-even, max/min, revenue, cost, profit  
fixed cost

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Panel 6

## Chapter 4: Exponential and Logarithm Functions

Recall: Rules of Exponents

$$b^x = \underbrace{b \cdot b \cdot b \cdots b}_{x\text{-times}}, \quad b^0 = 1$$

$$b^x \cdot b^y = b^{x+y} \quad (\text{same base - powers add})$$

$$(b \cdot c)^x = b^x c^x$$

*show it*  $\rightarrow b^{-x} = \frac{1}{b^x}$  (negative exp - flip it)

$$(b^x)^y = b^{xy} \quad (\text{powers to powers - multiply})$$

$$b^{1/2} = \sqrt{b}, \quad b^{1/n} = \sqrt[n]{b}$$

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Panel 7

Example: Simplify the following expressions:

$$\frac{x^2 y^3}{(x^4) y} = \frac{y^2}{x^2} \left( = x^2 y^3 \cdot x^{-4} y^{-1} = x^{2-4} y^{3-1} = x^{-2} y^2 = \frac{y^2}{x^2} \right)$$

$$\frac{x^2 + x^3 y^2}{\sqrt{x} y} = \frac{(x^2)(1 + xy^2)}{(x^{1/2}) y} = x^2 x^{-1/2} \frac{1 + xy^2}{y} = x^{3/2} \left( \frac{1 + xy^2}{y} \right)$$

$$\frac{(x^2)^2 (y^{-3})^2}{(\sqrt[4]{x^3}) (y^{-5})} = \frac{(x^2)^2 y^6}{y^3 (x^3)^{1/4}} = \frac{y^2 x^2}{x^{3/4}} = y^2 x^{2 - 3/4} = y^2 x^{5/4}$$

$$\frac{a^2 - b^2}{(a+b)^2 a^2} = \frac{(a+b)(a-b)}{(a+b)^2 a^2} = \frac{a-b}{(a+b)a^2}$$

Panel 8

### Exponential and Logarithm Functions

Def:  $f(x) = 5^x$ ,  $5 > 0$ , is exponential function with base 5.

Ex: Number of bacteria after  $t$  minutes is given by

$$N(t) = 300 \left( \frac{4}{3} \right)^t$$

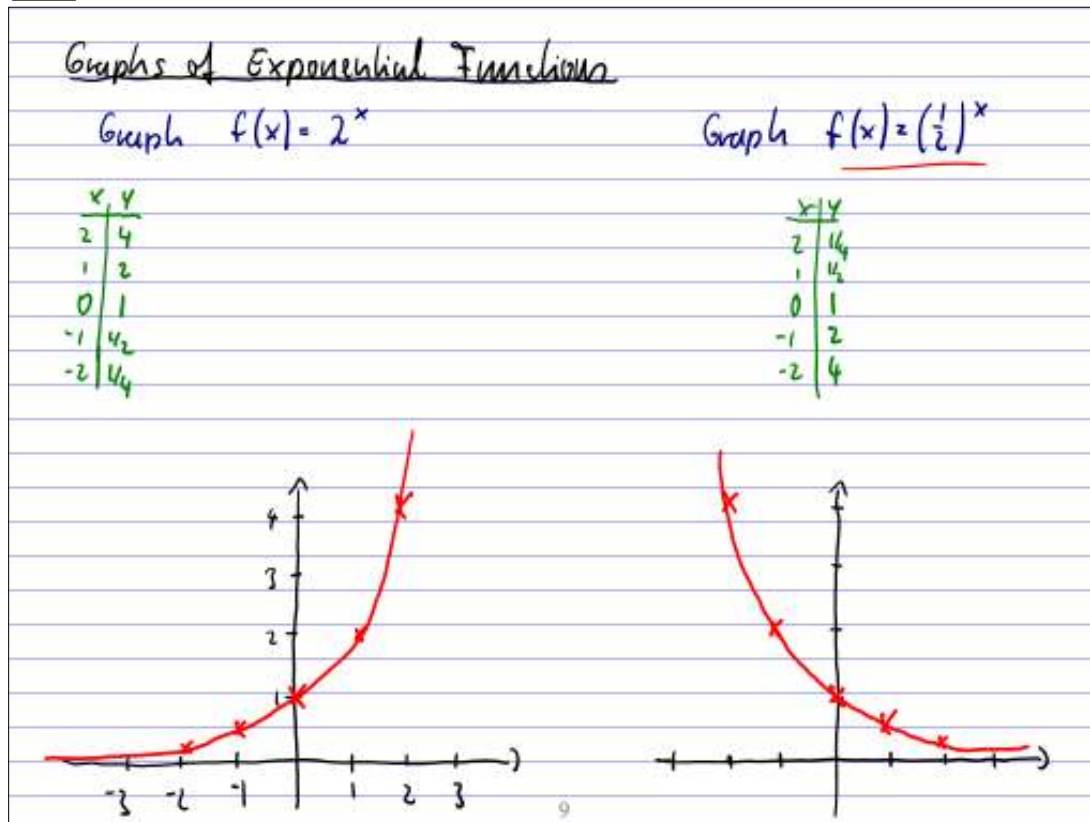
a) How many bacteria are present initially?

$$N(0) = 300 \left( \frac{4}{3} \right)^0 = \underline{300}$$

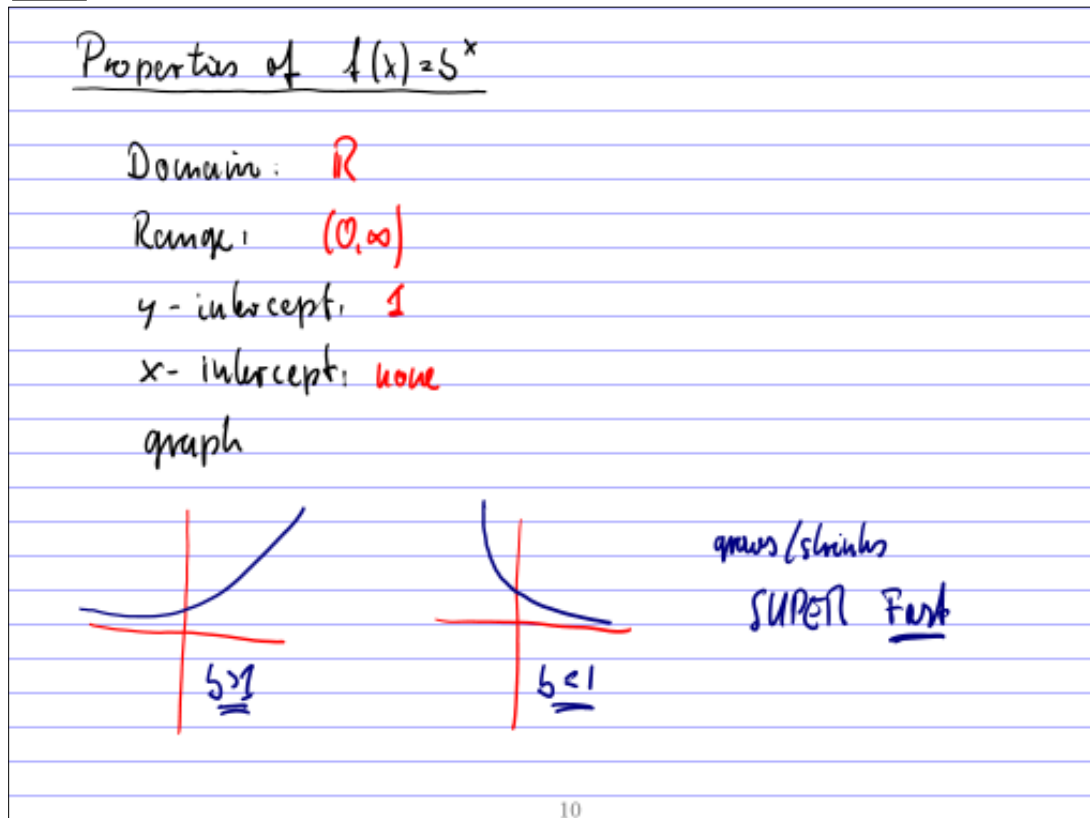
b) How many after 3 minutes?

$$N(3) = 300 \left( \frac{4}{3} \right)^3 = \underline{\underline{300 \frac{64}{27}}}$$

Panel 9



Panel 10



Panel 11

Exp. function is growing super-fast:

A sultan wanted to reward his wise man for his services and asked: "what do you want?"

The wise man thought briefly, then said: "Put one grain of rice on a chess board field and double it for each subsequent field".

$1=2^0$	$2=2^1$	$4=2^2$
$8=2^3$	$16=2^4$	$32=2^5$
$64=2^6$	$128=2^7$	$256=2^8$

chess board has 64 squares.

$$\Rightarrow 2^{63} =$$

9223372036854775808

more rice than ever produced

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Panel 12

### Application: Compound Interest

Suppose \$100 is invested at 5% interest, compounded annually. Final total amount after 2 years:

$$\text{year 0: } \underline{100} \quad \text{year 1: } 100 + 0.05 \cdot 100 = 105 \quad \text{year 2: } 105 + 0.05 \cdot 105 = \underline{110.25}$$

In general, principle  $P$ , rate  $r$ ,

year 0:  $P$

$$1: P + rP = \underline{P(1+r)}$$

$$2: \underline{P(1+r)} + r \underline{P(1+r)} = P(1+r)(1+r) = \underline{P(1+r)^2}$$

$$\underline{3: P(1+r)^2 + r P(1+r)^2 = P(1+r)^2(1+r) = \underline{P(1+r)^3}}$$

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Panel 13

The compounded amount  $S$  of a principle  $P$  at the end of  $n$  interest periods at a rate  $r$  per period is

$$S(n) = P \cdot (1+r)^n \quad \text{- exp. function}$$

Ex: \$1000 invested over 10 years at 6% annually:  
no compounding = 1600

$$S(10) = 1000(1+0.06)^{10} = \underline{1790.85}$$

\$1000 over 10 years at 6% annually, compounded monthly

$$1000 \left(1 + \frac{0.06}{12}\right)^{120} = \underline{1819.50}$$

\$1000 over 10 years at 6% annually, compounded daily:

$$1000 \left(1 + \frac{0.06}{365}\right)^{3650} = \underline{\$1922.03}$$

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Panel 14

Thought Experiment: Invest \$1 at 100% for one year, compounded  $n$ -times per year:

next time

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Panel 15

## The Natural Exponential Function

Define the number  $e = 2.7182\dots$  (Euler's number)

$f(x) = e^x = (2.7182\dots)^x$  is natural exp. function

$$f(0) = 1$$

$$f(1) = 2.7182\dots$$

$$f(2) = 7.389\dots$$

$$f(-3) = 0.049$$

$$f\left(\frac{1}{2}\right) = 1.641$$

