Panel 1

Last Time

Linear functions and their graphs

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y = mx + b$$

$$y - y_1 = m(x - x_1)$$

x- intercept: set $y = 0$

y- intercept set $x = 0$

demand and supply functions

systems of linear equations < elimination / substitution

1

Panel 2

Solve by elimination or substitution?

$(-2) \cdot \quad 2x + 7y = 3$ ✓ elimination

$4x + 15y = 9$

$x = 3y$ ✓ subst.

$2x + 4y = 20$

no solution

$\cdot\ x + 5y = \cancel{2}\,1 \longrightarrow (2 - \cancel{5y}) + \cancel{5y} = \cancel{2}\,1$

$\frac{1}{2}x = 1 - \frac{5}{2}y \quad | \cdot 2 \qquad 2 = \cancel{2}\,1$ inf. solutions (same lines)

$\cdot\ x = 2 - 5y$

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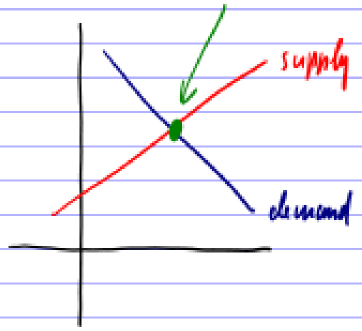
Panel 3

Application: Suppose for product Z we have

$$p = -\frac{1}{180} q + 12 \qquad \text{(demand)}$$

$$p = \frac{1}{300} q + 8 \qquad \text{(supply)}$$

The price point when supply and demand are the same is called equilibrium point.



$$-\frac{1}{180} q + 12 = \frac{1}{300} q + 8$$

$$4 = \left(\frac{1}{300} + \frac{1}{180}\right) q$$

$$4 = 0.00889\, q \Rightarrow q = \frac{4}{0.0089} \approx \underline{450}$$

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Panel 4

www.wolframalpha.com

our favorite search engine

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Panel 5

Final application: A chemical manufacturer wants to produce 500 liters of a 25% acid solution. She has solutions of 30% and 18% acidity in stock. How should they be mixed?

How much of each to use?

x = # of liters of 30%, y = # of liters of 18%

$$\underline{x} + y = 500$$

acidity: $0.3x + 0.18y = 0.25 \cdot 500$

Result:

$$x = \frac{875}{3} \text{ and } y = \frac{625}{3}$$

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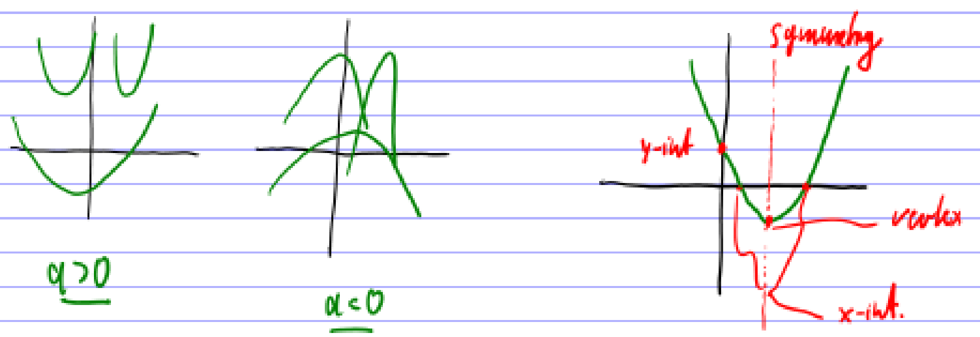
Panel 6

Quadratic Functions

$f(x) = ax^2 + bx + c$



$\underline{a > 0}$ $\underline{a < 0}$

$f(x) = 2x^2$ $f(x) = -3x^2$

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Panel 7

Facts about quadratic functions

$f(x) = ax^2 + bx + c$

opens up $(a > 0)$ or down $(a < 0)$

vertex: $x = -b/2a$, $y = f(-b/2a)$ substitute

y-int: $y = c$ easy

x-int: $0 = ax^2 + bx + c$ ↙ or factor

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

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Panel 8

Ex: Graph $f(x) = -x^2 - 4x + 12$

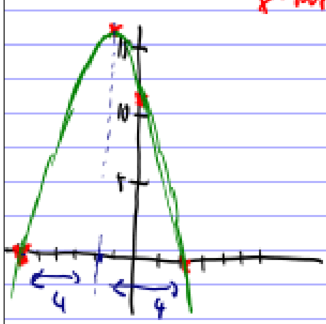
down

vertex: $x = 4/-2 = -2$, $y = 16$

y-int: $y = 12$

x-int: $0 = -x^2 - 4x + 12$

$= -(x^2 + 4x - 12) = -(x+6)(x-2) \Rightarrow x = -6, 2$



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Panel 9

Ex: Graph $g(x) = x^2 - 6x + 7$

vertex: $x = 6/2 = 3$, $y =$

x-int: $0 = x^2 - 6x + 7$

$$x = \frac{6 \pm \sqrt{36 - 28}}{2} = \frac{6 \pm \sqrt{8}}{2} < \begin{matrix} \# \\ \# \end{matrix}$$

Rest is HW

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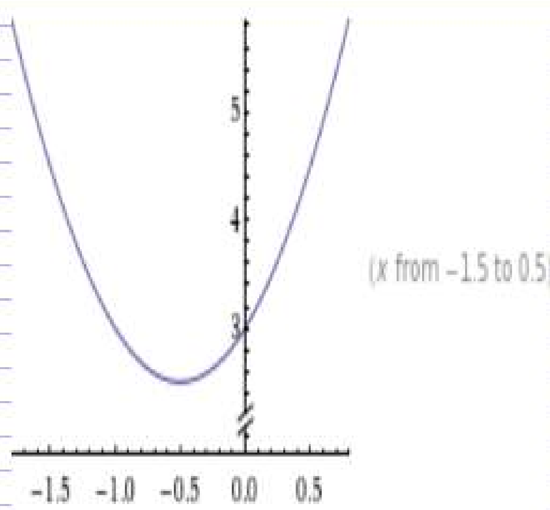
Panel 10

Ex: Graph $f(x) = 2x^2 + 2x + 3$ and find range



$\frac{-b \pm \sqrt{b^2 - 4ac}}{2}$ no solution in this case

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Panel 11

Ex: Suppose the demand for a product is
$p = 1000 - 2q$, q = # of units and p price
per unit if q units are demanded (weekly)
Max. Revenue.

$p = p(q) = 1000 - 2q \Rightarrow R(q) = p \cdot q = (1000 - 2q) \cdot q =$
$= 1000q - 2q^2$

parabola going down with vertex is max. In this case

$q = +\frac{1000}{+4} = \underline{\underline{250}}$

Produce $q = 250$ units for max. Revenue of $R(250) = 125000$

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Panel 12

Cost, Revenue, Profit

Cost of producing q items is comprised of the
y_{FC} fixed cost (for producing zero items) plus
y_{VC} variable cost (cost depending on q)

$C(q) = y_{TC} = y_{VC} + y_{FC}$

Revenue: $R(q) = p \cdot q = p(q) \cdot p$

Profit: $P(q) = R(q) - C(q)$

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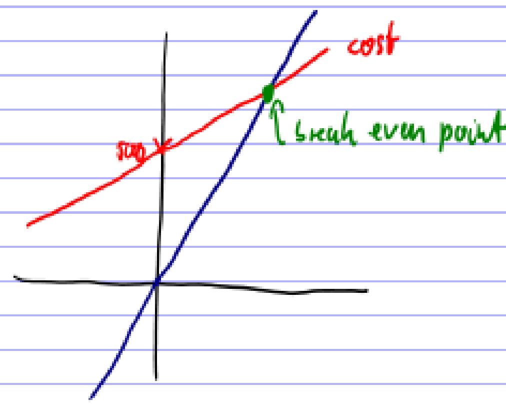
Panel 13

Example: A company sells a product for \$8 per unit. Fixed costs are \$5000 and variable costs are \$3. Graph cost and revenue functions.

$C(q) = 5000 + 3q$

$R(q) = 8q$



Break even point:

$R(q) = C(q)$ or

$P(q) = 0$

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Panel 14

Break-Even Point:

$C(q) = 3q + 5000$

$R(q) = 8q$

Break-even: $3q + 5000 = 8q$

$5000 = 5q$

$\underline{1000 = q}$

$P(1000) = 0$

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Panel 15

Ex: Suppose company XYZ has

$r(q) = 100\sqrt{q}$ and

$c(q) = 2q + 1200$

Find fixed cost, variable cost, and break-even point.

$C(0) = 1200$, $\$2.00$

BE point: $100\sqrt{q} = 2q + 1200$ $\quad | ( )^2$ HW (2 ways)

$$(100\sqrt{q})^2 = (2q + 1200)^2$$

$$10000q = 4q^2 + 4800q + 1440000$$

$$0 = 4q^2 - 5200q + 1440000$$

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