Panel 1

Last Time

Linear functions and their graphs

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y = mx + b$$

$$y - y_1 = m(x - x_1)$$

x- intercept: set $y = 0$

y- intercept set $x = 0$

demand and supply functions

systems of linear equations elimination / substitution

1

Panel 2

Solve by elimination or substitution?

$(-2) \cdot | \; 2x + 7y = 3$ elimination

$4x + 15y = 9$ 1 answer

$x = 3y$ substitution

$2x + 4y = 20$

$x + 5y = \cancel{2}1$ $\Rightarrow (2 - 5y) + 5y = \cancel{2}1$ no answer (false)

$\frac{1}{2}x = 1 - \frac{5}{2}y \quad | \cdot 2$ $2 = \cancel{2}1$

inf. many answers

$x = 2 - 5y$

2

Panel 3

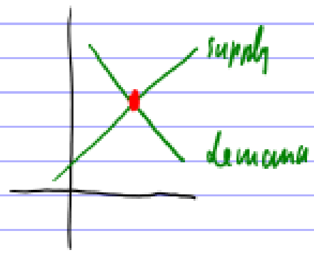
Application: Suppose for product X we have

$$p = -\frac{1}{180} q + 12 \quad \text{(demand)}$$

$$p = \frac{1}{300} q + 8 \quad \text{(supply)}$$

The point where supply + demand are the same.

Equilibrium Point!



$$\frac{1}{300} q + 8 = -\frac{1}{180} q + 12$$

$$q\left(\frac{1}{300} + \frac{1}{180}\right) = 4$$

$$0.0089\, q = 4$$

$$q = 4 \cdot \frac{1}{0.0089} \approx \underline{450}$$

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Panel 4

www.wolframalpha.com

use website for advanced calculations!

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Panel 5

Final application: A chemical manufacturer wants to produce 500 liters of a 25% acid solution. She has solutions of (A) 30% and (B) 18% acidity in stock. How should they be mixed?

How many liters of type A and type B liquid to use

x = # of liters of 30%, y = # of liters of 18%

$$x + y = 500$$

acidity $\quad 0.3x + 0.18y = 0.25 \cdot 500$

125

Result:

$$x = \frac{875}{3} \text{ and } y = \frac{625}{3}$$

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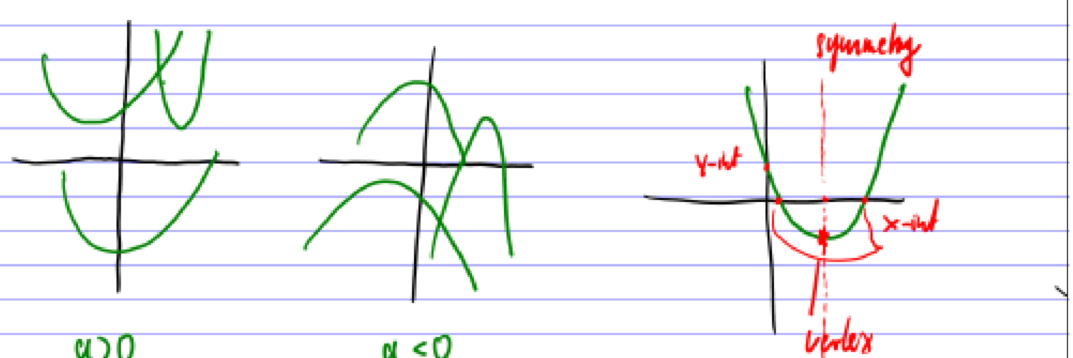
Panel 6

Quadratic Functions

$f(x) = ax^2 + bx + c$ (quadratic function)



$a > 0$ $\qquad$ $a < 0$

$f(x) = 2x^2$ $\qquad$ $f(x) = -x^2$

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Panel 7

Facts about quadratic functions : ax^2+bx+c

opens up $(a>0)$ or down $(a<0)$

vertex : $x = -b/2a$, y = subst x into $f(x)$

y-int : (set $x=0$) $y=c$

x-int : $0 = ax^2+bx+c$ ← or factor

$$\boxed{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$

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Panel 8

Ex: Graph $f(x) = -x^2 - 4x + 12$

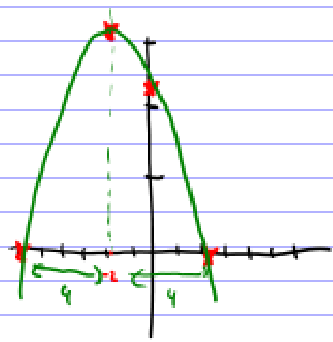
down

vertex : $x = \frac{4}{-2} = -2$, $y = 16$

y-int : $y = 12$

x-int : $0 = -x^2 - 4x + 12$

$= -(x^2+4x-12) = -(x+6)(x-2) \Rightarrow x = -6, 2$



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Panel 9

Ex: Graph $g(x) = x^2 - 6x + 7$

opens up

vertex: $x = \frac{6}{2} = 3$, $y = -2$

y-int: $y = 7$

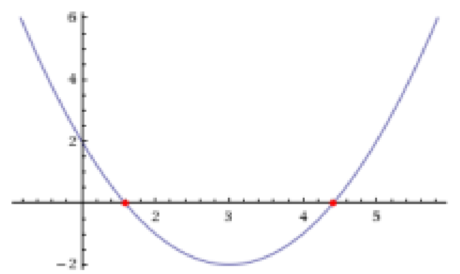
x-int: hard

Results:

$x = 3 - \sqrt{2} \approx 1.58579$

$x = 3 + \sqrt{2} \approx 4.41421$

Root plot:



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Panel 10

Ex: Graph $f(x) = 2x^2 + 2x + 3$ and find range

HW

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Panel 11

Ex: Suppose the demand for a product is

$p = \underline{1000 - 2q}$, q = # of units and p price

per unit if q units are demanded (weekly)

Find max revenue!

$p(q) = 1000 - 2q \Rightarrow R(q) = p \cdot q = (1000 - 2q) \cdot q$

$= 1000q - 2q^2$

max rev. is at vertex of parabola going down.

vertex $q = -b/2a = \frac{-1000}{-4} = \underline{\underline{250}}$

Produce $q = 250$ units for max revenue of $R(\underline{250}) = \#$

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Panel 12

Cost, Revenue, Profit

Cost of producing q-items is comprised of

fixed cost (cost for producing 0 items) plus

variable cost (cost depending on q)

Total cost: $y_{TC} = y_{VC} + y_{FC}$

total variable fixed

Revenue: $R(q) = p \cdot q$

Profit: $P(q) = R(q) - C(q)$

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Panel 13

Example: A company sells a product for \$8 per unit. Fixed costs are \$5000 and variable costs are \$3. Graph cost and revenue functions.

$C(q) = Y_{VC} + Y_{FC} =$

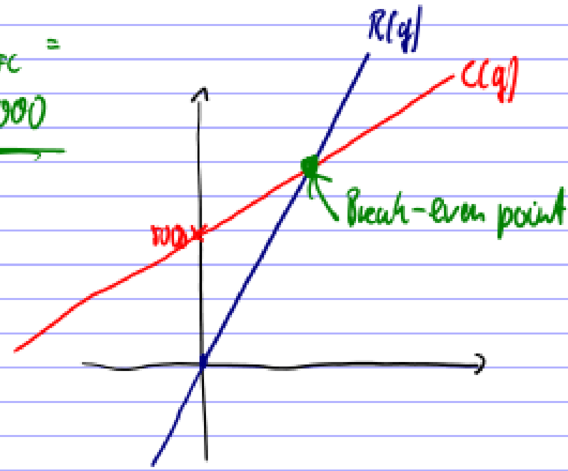
$= 3q + 5000$

$R(q) = 8q$

Break-Even Point is

$R(q) = C(q)$ or

$P(q) = 0$



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Panel 14

Break-Even Point:

$8q = 3q + 5000$

$5q = 5000$

$q = 1000 \quad \Rightarrow P(1000) = 0$

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Panel 15

Ex: Suppose company XYZ has

$r(q) = 100\sqrt{q}$ and

$c(q) = 2q + 1200$

Find fixed cost, variable cost, and break-even point.

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