Math 1303: Practice Exam 1

This practice exam has many more questions that the real exam. The real exam will have 10 questions (some multi-part) of the type covered in this practice exam and in the homework and quizzes. If you have any questions, please email me.

1. Find the domain of the following functions:
   a. $f(x) = \frac{x}{x^2-6x+5}$ $x^2-6x+5=0 \iff (x-5)(x-1)=0 \Rightarrow x=5,1$ Domain: $\mathbb{R}$ except $x=5,1$
   b. $f(x) = \frac{x-6}{\sqrt{2x-3}}$ $2x-3>0$, $2x>3$ Domain: all $x > 3/2$
   c. $f(x) = \log_2(x)$ any $\log_b(x)$ has as domain all $x>0$
   d. d. $k(t) = \frac{2x^2-3x}{e^x}$ e^x is never zero, so Domain is all $\mathbb{R}$
   e. $f(x) = \log_2(x-1)$ $x-1>0 \Rightarrow x>1 \Rightarrow$ Domain: all $x>1$

2. Suppose $f(x) = 2x^2 - 3$ and $g(x) = \frac{1}{x^2-1}$. Find the following quantities:
   a. $(f \circ g)(x)$ $= f(g(x)) = f\left(\frac{1}{x^2-1}\right) = 2\left(\frac{1}{x^2-1}\right)^2 - 3$
   b. $(g \circ f)(x)$ $= g(f(x)) = g(2x^2-3) = \frac{1}{(2x^2-3)^2-1}$
   c. $(f \circ f)(x)$ $= f(f(x)) = f(2x^2-3) = 2(2x^2-3)^2-3$
   d. $\frac{f(x)}{g(x)}$ $= \frac{2x^2-3}{1/x^2-1} = (2x^2-3)(x^2-1)$

3. Suppose $f(x) = 2x^2 - 3$. Compute

   $f(-2)$ $= 2(-2)^2-3 = 8-3 = 5$

$f(3t)$ $= 2(3t)^2 - 3 = 2 \cdot 9t^2 - 3 = \underline{\underline{18t^2 - 3}}$

$\frac{f(x+h)-f(x)}{h} = \frac{[2(x+h)^2 - 3] - [2x^2 - 3]}{h} = \frac{2(\cancel{x^2} + 2xh + h^2) \cancel{-3} - \cancel{2x^2} \cancel{+3}}{h} = \frac{4xh + 2h^2}{h} = \frac{\cancel{h}(4x+2h)}{\cancel{h}}$

$= \underline{\underline{4x + 2h}}$

Do the same for the function $f(x) = -3x + 6$.

$f(-2)$,

$= -3(-2) + 6 = 12$

$f(3t)$, $= -3(3t) + 6 = \underline{-9t + 6}$

$\frac{f(x+h)-f(x)}{h} = \frac{[-3(x+h)+6] - [-3x+6]}{h} = \frac{\cancel{-3x} - 3h \cancel{+6} \cancel{+3x} \cancel{-6}}{h} = \frac{-3\cancel{h}}{\cancel{h}} = \underline{\underline{-3}}$

4. Let $h(x) = \begin{cases} 1 - x & if \;\; x \geq 0 \\ 3x - 2 & if \;\; x < 0 \end{cases}$ and $g(x) = \begin{cases} x^2 & if \;\; x < 0 \\ 2 - 2x & if \; 0 \leq x \leq 1 \\ 2 - x^2 & if \;\; 1 < x \end{cases}$

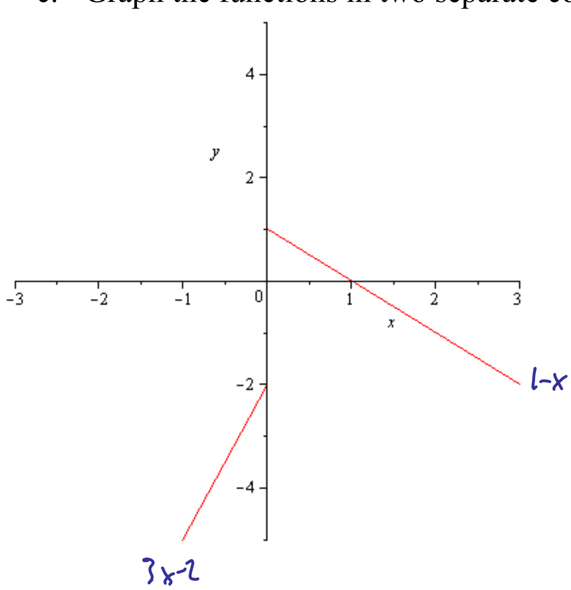
a. Find $h(-2)$ $= -6 - 2 = \underline{\underline{-8}}$

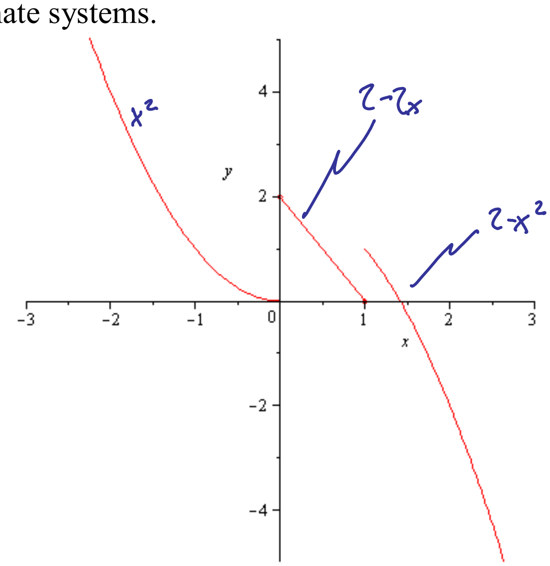
$g(-2)$ $= (-2)^2 = \underline{\underline{4}}$

b. Find $h(0)$ $= 1 - 0 = \underline{\underline{1}}$

$g(1)$ $= 2 - 2 \cdot 1 = \underline{\underline{0}}$

c. Graph the functions in two separate coordinate systems.





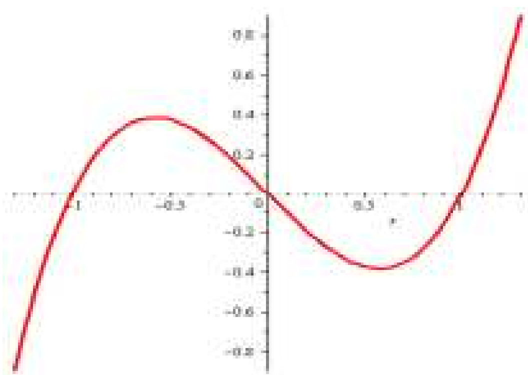
5. Decide whether the following functions are even, odd, or neither:

a) $f(x) = 2x^4 - x^2 + 1$ $\quad f(-x) = 2(-x)^4 - (-x^2) + 1 = 2x^4 - x^2 + 1 = f(x)$: even

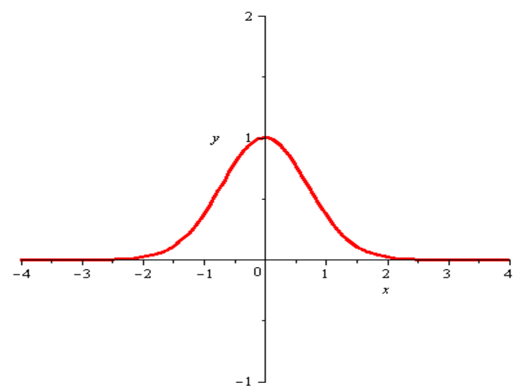
b) $g(x) = \frac{x}{x^2-1}$ $\quad g(-x) = \frac{-x}{(-x)^2-1} = \frac{-x}{x^2-1} = -\frac{x}{x^2-1} = -g(x) \Rightarrow$ odd

c) $h(x) = (x^2-1)(x^3+1)$ $\quad h(-x) = ((-x)^2-1)((-x)^3+1) = (x^2-1)(-x^3+1) \Rightarrow$ neither

d) $k(x) = 3x\, e^{x2}$ $\quad k(-x) = -3x\, e^{(-x)^2} = -3x\, e^{x^2} = -k(x) \Rightarrow$ odd

e) 

symmetric about origin
odd

f) 

symmetric about y-axis
even

6. Find the equation of a line satisfying the given conditions

a. through $(-1,2)$ and $(2,3)$

$m = \frac{3-2}{2-(-1)} = \frac{1}{3} \Rightarrow y - 3 = \frac{1}{3}(x-2)$

b. through (3, 4) and (-2, 4)

$m = \frac{4-4}{-2-3} = 0 \Rightarrow y = 4$

c. through the point $(3,1)$ parallel to the line $6x - 3y = 6$ $\Rightarrow -3y = 6 - 6x \Rightarrow y = -2 + 2x$ slope is 2

$\Rightarrow y - 1 = 2(x-3)$

d. through the point (2, 1) perpendicular to the line $y = 3x - 1$ slope is 3

$\Rightarrow y - 1 = -\frac{1}{3}(x-2)$

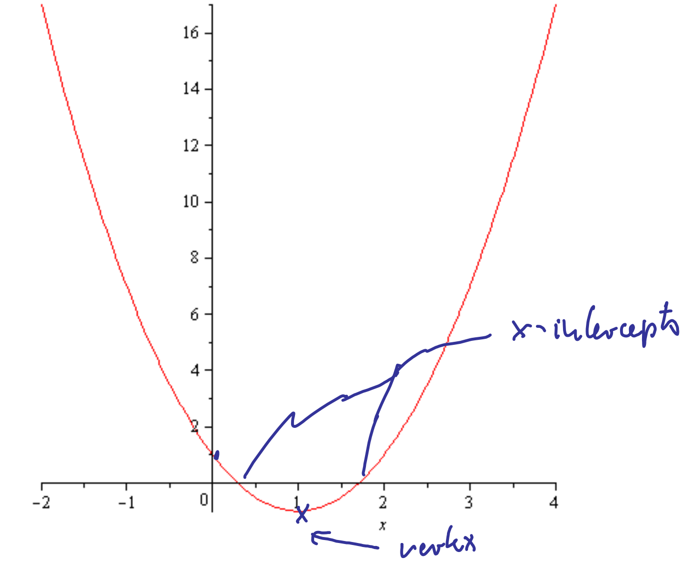
e. with x-intercept 2 and y-intercept 4

i.e. through $(2,0)$ and $(0,4)$ $\Rightarrow m = \frac{4-0}{0-2} = -2$

$\Rightarrow y = -2x + 4$

7. Find the vertex, x-intercepts, y-intercepts and graphs for

a) $y = 2x^2 - 4x + 1$



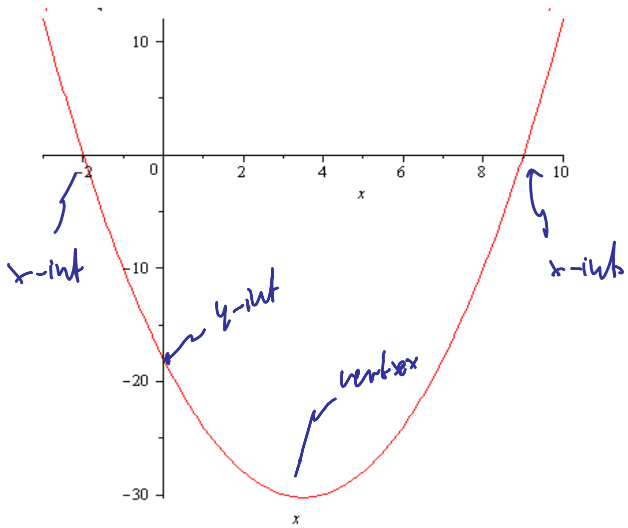
vertex: $x = -\frac{b}{2a} = \frac{4}{4} = 1$ $\quad$ $\underline{(1,-1)}$

$y = 2-4+1 = -1$

x-intercepts: $x = \frac{4 \pm \sqrt{16-8}}{4} = \underline{1.707}$ and $\underline{0.293}$

y-intercept: $y = 1$

b) $y = x^2 - 7x - 18$



vertex: $x = -\frac{b}{2a} = \frac{7}{2} = 3.5$ $\quad$ $\underline{(3.5, 30.25)}$

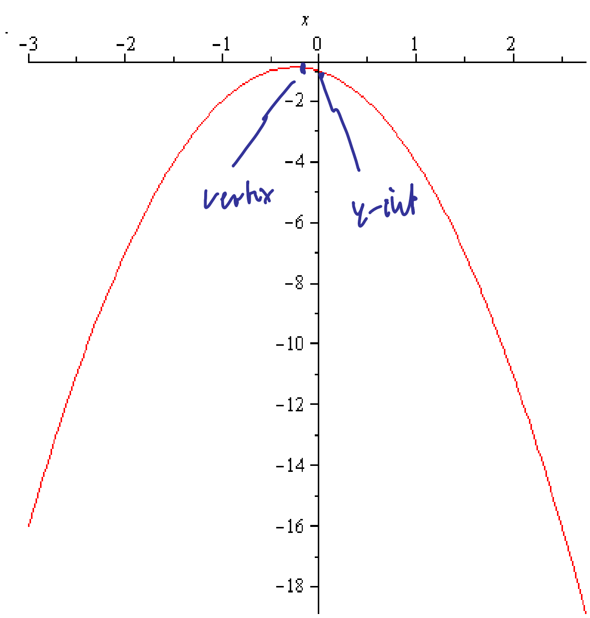
$y = 30.25$

x-intercepts: $x^2 - 7x - 18 = (x-9)(x+2) = 0$

$\underline{x=-2}$, $\underline{x=9}$

y-intercept: $\underline{y = -18}$

c) $y = -2x^2 - x - 1$



vertex: $x = -\frac{b}{2a} = \frac{1}{4} = -0.25$ $\quad$ $\underline{(-0.25, -0.6875)}$

$y = -0.6875$

x-intercepts: $x = \frac{1 \pm \sqrt{1-8}}{4} = \underline{\text{none}}$

y-intercept: $\underline{y = -1}$

8. Solve the systems of equations, if possible

$$2x - y = 6$$
$$3x + 2y = 5$$

$1 \cdot 2 \Rightarrow$

$$4x - 2y = 12$$
$$3x + 2y = 5$$
$$7x = 17$$

$\Rightarrow \underline{\underline{x = \frac{17}{7}}}, \; \underline{\underline{y = -\frac{9}{7}}}$

$$8x - 4y = 7$$
$$y = 2x - 4$$

$8x - 4(2x - 4) = 7$

$8x - 8x + 16 = 7 \Rightarrow 16 = 7$ false so <u>no solution</u>

$$2x - y = 3$$
$$-4x + 2y = 8$$

$1 \cdot 2$

$$4x - 2y = 6$$
$$-4x + 2y = 8$$
$$0 = 14$$

false so <u>no solution</u>

$$x + y + z = 0$$
$$x - y + z = 1$$
$$-x + y - z = 2$$

$\Rightarrow$

$$x + y + z = 0$$
$$2y = -1$$
$$-x + y - z = 2$$

$\Rightarrow$

$$x + y + z = 0$$
$$2y = -1$$
$$2y = 2$$

$\Rightarrow$

$$x + y + z = 0$$
$$2y = -1$$
$$0 = -3$$

false so no solution

~~(This is unfortunate. In exam there likely will be a solution!)~~

9. A company makes 3 types of furniture: chairs, rockers, and chaise lounges. Each require wood, plastic, and aluminum in the quantities shown in the table below. The company stocks 8 units of wood, 10 units of plastic, and 12 units of aluminum. How many chairs, rockers, and chaise lounges should the company produce, assuming they use all of their inventory?

	Wood	**Plastic**	**Aluminum**
Chair	1 unit	1 unit	2 units
Rocker	1 unit	1 unit	1 unit
Chaise Lounge	1 unit	2 units	2 units

Hint: If *x* is the number of chairs, *y* the number of rockers, and *z* the number of chaise lounges, then this situation can be modeled by the system of equations:

$$x + y + z = 8$$
$$x + y + 2z = 10$$
$$2x + y + 2z = 12$$

$\Rightarrow$

$$x + y + z = 8$$
$$z = 2$$
$$2x + y + 2z = 12$$

$1 \cdot (-2)$

$$x + y + z = 8$$
$$z = 2$$
$$-y = -4$$

not exactly the right form but we can see that $\underline{\underline{y = 4}}$, $\underline{\underline{z = 2}}$ and therefore $\underline{\underline{x = 2}}$

Thus: Produce 2 chairs, 4 rockers, and 2 chaise lounges

Profit makes no sense here, I meant revenue! Sorry!

10. Suppose a ~~profit~~ Revenue function is given $\cancel{p}(q)$ R $= 200q$ while the cost function is $C(q) = 250 + 100q$. Find the fixed cost and the break-even point(s).

Fixed cost is $C(0) = \underline{250}$

Break-even: $R(q) = C(q) \iff 200q = 250 + 100q \Rightarrow 100q = 250 \Rightarrow \underline{q = 2.5}$

Break-even at production level $\underline{q = 2.5}$

$100p + q - 1200 = 0$ (sorry again)

11. If the supply and demand functions of a product are $120p - q - 240 = 0$ and $100p_q - 1200$, respectively, find the equilibrium price.

$120p - q - 240 = 0 \Rightarrow q = 120p - 240$
$100p + q - 1200 = 0 \quad q = -100p + 1200$

$\Rightarrow 120p - 240 = -100p + 1200$
$220p = 1440$

Sorry again

$\Rightarrow \underline{p = 6.54}, \quad \underline{q = 545.45}$

12. Suppose a demand and supply equation are, respectively, $5p - q = 10$ and $2p^2 - q = 8$. Find the equilibrium price (there may be more than one)

Finally a good one with no typos!

$5p - q = 10 \Rightarrow 5p - 10 = q$
$2p^2 - q = 8 \quad 2p^2 - 8 = q$

$\Rightarrow 2p^2 - 8 = 5p - 10 \iff 2p^2 - 5p + 2 = 0$

$p = \frac{5 \pm \sqrt{25 - 16}}{4} = \frac{5 \pm 3}{4} = \frac{1}{2}$ and $2 \Rightarrow$ $\underline{p = 1/2}, \underline{q = -\frac{15}{2}}$; $\underline{p = 2}, \underline{q = 0}$

13. The demand function for a product is $p(q) = 200 - 2q$ where p is the price in dollars per unit when q units are demanded. Find the level of production that maximizes the manufacturer's revenue.

$p(q) = 200 - 2q$ demand $\Rightarrow R(q) = q(200 - 2q) = 200q - 2q^2$

parabola opening down $\Rightarrow$ vertex is max. vertex $q = \frac{-200}{-4} = \underline{50}$

$R = 50 \cdot 100 = 5000$

Produce $\underline{50 \text{ units}}$ for max revenue $\underline{\$5000}$

14. A manufacturer sells all units produced. What is the break-even point if the product is sold at \$16 per unit, fixed cost is \$10,000, and variable cost is $y_{vc} = 8q$, where q is the number of units produced.

$R(q) = 16q$

$C(q) = 8q + 10000$

$\Rightarrow$ Break-even: $16q = 8q + 10000 \Rightarrow q = \frac{10000}{8} = \underline{1250}$

Produce 1250 units to break even

15. A manufacturer sells a product at \$8.35 per unit, selling all produced. The fixed cost is \$2,116 and the variable cost is \$7.20 per unit. At what level of production will the break-even point occur?

$R(q) = 8.35q$

$C(q) = 7.2q + 2116$

$\Rightarrow$ Break-even: $8.35q = 7.2q + 2116$

$\Rightarrow \underline{q} = \frac{2116}{1.15} = \underline{1840}$ is break-even production

16. Suppose you invest \$250 at 4% interest, compounded monthly. How much money will you have after 3 years? How much would you have if there was no compounding at all?

$S = P(1+r)^n = 250\left(1 + \frac{0.04}{12}\right)^{3 \cdot 12} = 250(1.00333)^{36} = \underline{281.82}$

no compounding: $250 \cdot 0.04 \cdot 3 = 30$ interest $\Rightarrow 250 + 30 = \underline{280}$

17. Suppose you want to invest \$5,000 at 5% interest for 10 years. Bank A offers quarterly compounding, Bank ~~A~~ B compounds weekly. Where would you invest your money and how much money would the difference be between bank A and B after 10 years?

Bank A: $5000\left(1 + \frac{0.05}{4}\right)^{10 \cdot 4} = \underline{\$8218.10}$

Bank B: $5000\left(1 + \frac{0.05}{52}\right)^{520} = \underline{\$8241.63}$

Difference $\underline{\$23.53}$ (favor bank B)

18. Evaluate the following expressions:

a) $\log_5(125) = y$

$5^y = 125 \Rightarrow \underline{y = 3}$

b) $\log_3\left(\frac{1}{81}\right) = y$

$3^y = 1/81 = \frac{1}{3^4} = 3^{-4} \Rightarrow \underline{y = -4}$

c) $\log_4(2) = y$

$4^y = 2 \Rightarrow \underline{y = 1/2}$

d) $\log_{\frac{1}{3}}(9) = y$ $\frac{1}{3}^{y} = 9 \Rightarrow \underline{\underline{y = -2}}$

19. Solve for x:

a) $\log_2(x) = 6$ $\Leftrightarrow 2^6 = \underline{\underline{x = 64}}$

b) $\log(6x - 2) = 2$ $\Leftrightarrow 10^2 = 6x - 2 \Rightarrow 100 = 6x - 2 \Rightarrow 102 = 6x \Rightarrow \underline{x} = \frac{102}{6} = \underline{17}$

c) $3^{4x} = 9^{x+1}$ $\Leftrightarrow 3^{4x} = (3^2)^{x+1} \Rightarrow 3^{4x} = 3^{2x+2} \mid \log_3() \Rightarrow 4x = 2x + 2 \Rightarrow \underline{\underline{x = 1}}$

d) $4^{3-x} = \frac{1}{16}$ $\Leftrightarrow 4^{3-x} = 4^{-2} \Rightarrow 3 - x = -2 \Rightarrow \underline{\underline{x = 5}}$

e) $e^{3x} = 14$ $\mid \ln()$

$\cancel{\ln}(\cancel{e}^{3x}) = \ln(14) \Rightarrow 3x = \ln(14) \Rightarrow \underline{\underline{x}} = \underline{\underline{\frac{\ln(14)}{3}}} = \underline{\underline{0.8797}}$

20. The population of a fast-growing town in the south is modeled by the equation $P(t) = 7{,}000\, e^{0.09 \cdot t}$ where t is the number of years past 1990.

a. What was the population of the town in 1990?

$P(0) = 7000 \cdot e^0 = \underline{\underline{7000}}$

b. What will the population be in 2030? $P(40) = 7000\, e^{0.09 \cdot 40} = 7000 e^{3.6} \approx \underline{\underline{256{,}187.}}$

c. When, approximately, will the population double in size?

$P(t) = 14000 \Rightarrow 7000 e^{0.09t} = 14000 \Rightarrow e^{0.09t} = 2$

$\Rightarrow 0.09t = \ln(2) \Rightarrow t = \frac{\ln(2)}{0.09} = \underline{\underline{7.7}}$

Pop. will double every 7.7 years

21. A radioactive substance decays according to $N(t) = 10\,e^{-0.14t}$ where N is the number of mg present after t hours. How much of the substance s initially present? How much is present after 5 hours? After how many hours is 0.1 mg remaining?

$N(0) = 10$ mg initially.

After 5 hours: $N(5) = 10e^{-0.14\cdot 5} = 4.96$ mg

$N(t) = 0.1 \Leftrightarrow 10e^{-0.14t} = 0.1 \Rightarrow e^{-0.14t} = 0.01 \Rightarrow -0.14t = \ln(0.01)$

$\Rightarrow t = -\dfrac{\ln(0.01)}{0.14} = 19.2$

After 19.2 hours 0.1mg remains

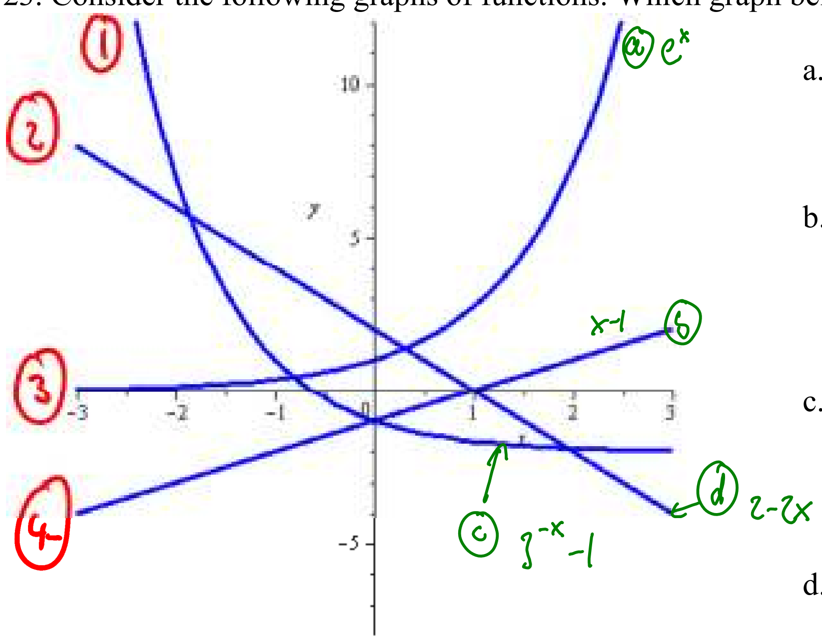
22. If I invest $2,500 at 6.5% interest, compounded monthly, for how many years should I invest it to reach my goal of having $20,000?

$20000 = 2500\left(1 + \frac{0.065}{12}\right)^{12\cdot t} \Rightarrow 8 = 1.00542^{12\cdot t}$

$\Rightarrow \ln(8) = \ln\left(1.00542^{12\cdot t}\right) = 12\cdot t\cdot \ln(1.00542)$

$\Rightarrow t = \dfrac{\ln(8)}{12\cdot \ln(1.00542)} = \dfrac{2.079}{0.065} \approx 32$ years

23. Consider the following graphs of functions. Which graph belongs to which function?



a. $f(x) = e^x$

b. $g(x) = x - 1$

c. $h(x) = 3^{-x} - 1$

d. $k(x) = 2 - 2x$