

TECHNOLOGY

Two functions can be combined by using a graphing calculator. Consider the functions

$$f(x) = 2x + 1 \quad \text{and} \quad g(x) = x^2$$

which we enter as Y_1 and Y_2 , as shown in Figure 2.3. The sum of f and g is given by $Y_3 = Y_1 + Y_2$ and the composite $f \circ g$ by $Y_4 = Y_1(Y_2)$. For example, $f(g(3))$ is obtained by evaluating Y_4 at 3.



FIGURE 2.3 Y_3 and Y_4 are combinations of Y_1 and Y_2 .

PROBLEMS 2.3

1. If $f(x) = x + 3$ and $g(x) = x + 5$, find the following.

- (a) $(f + g)(x)$
- (b) $(f + g)(0)$
- (c) $(f - g)(x)$
- (d) $(fg)(x)$
- (e) $(fg)(-2)$
- (f) $\frac{f}{g}(x)$
- (g) $(f \circ g)(x)$
- (h) $(f \circ g)(3)$
- (i) $(g \circ f)(x)$
- (j) $(g \circ f)(3)$

2. If $f(x) = 2x$ and $g(x) = 6 + x$, find the following.

- (a) $(f + g)(x)$
- (b) $(f - g)(x)$
- (c) $(f - g)(4)$
- (d) $(fg)(x)$
- (e) $\frac{f}{g}(x)$
- (f) $\frac{f}{g}(2)$
- (g) $(f \circ g)(x)$
- (h) $(g \circ f)(x)$
- (i) $(g \circ f)(2)$

3. If $f(x) = x^2 - 1$ and $g(x) = x^2 + x$, find the following.

- (a) $(f + g)(x)$
- (b) $(f - g)(x)$
- (c) $(f - g)\left(-\frac{1}{2}\right)$
- (d) $(fg)(x)$
- (e) $\frac{f}{g}(x)$
- (f) $\frac{f}{g}\left(-\frac{1}{2}\right)$
- (g) $(f \circ g)(x)$
- (h) $(g \circ f)(x)$
- (i) $(g \circ f)(-3)$

4. If $f(x) = x^2 + 1$ and $g(x) = 5$, find the following.

- (a) $(f + g)(x)$
- (b) $(f + g)\left(\frac{7}{5}\right)$
- (c) $(f - g)(x)$
- (d) $(fg)(x)$
- (e) $(fg)(7)$
- (f) $\frac{f}{g}(x)$
- (g) $(f \circ g)(x)$
- (h) $(f \circ g)(12,003)$
- (i) $(g \circ f)(x)$

5. If $f(x) = 3x^2 + 6$ and $g(x) = 4 - 2x$, find $f(g(2))$ and $g(f(2))$.

6. If $f(p) = \frac{4}{p}$ and $g(p) = \frac{p-2}{3}$, find both $(f \circ g)(p)$ and $(g \circ f)(p)$.

7. If $F(t) = t^2 + 7t + 1$ and $G(t) = \frac{2}{t-1}$, find $(F \circ G)(t)$ and $(G \circ F)(t)$.

8. If $F(t) = \sqrt{t}$ and $G(t) = 2t^2 - 2t + 1$, find $(F \circ G)(t)$ and $(G \circ F)(t)$.

In Problems 11–16, find functions f and g such that $h(x) = f(g(x))$.

11. $h(x) = 11x - 7$

12. $h(x) = \sqrt{x^2 - 2}$

13. $h(x) = \frac{3}{x^2 + x + 1}$

14. $h(x) = (9x^2 - 5x)^4 - (9x^2 - 5x)^2 + 11$

15. $h(x) = \sqrt{\frac{x^2 - 1}{x + 3}}$

16. $h(x) = \frac{2 - (3x - 5)}{(3x - 5)^2 + 2}$

17. **Profit** A coffeehouse sells a pound of coffee for \$9.75. Expenses are \$4500 each month, plus \$4.25 for each pound of coffee sold.

- (a) Write a function $r(x)$ for the total monthly revenue as a function of the number of pounds of coffee sold.
- (b) Write a function $e(x)$ for the total monthly expenses as a function of the number of pounds of coffee sold.
- (c) Write a function $(r - e)(x)$ for the total monthly profit as a function of the number of pounds of coffee sold.

18. **Geometry** Suppose the volume of a sphere is $v(x) = \frac{4}{3}\pi(3x - 1)^3$. Express v as a composite of two functions, and explain what each function represents.

19. **Business** A manufacturer determines that the total number of units of output per day, q , is a function of the number of employees, m , where

$$q = f(m) = \frac{(40m - m^2)}{4}$$

The total revenue r that is received for selling q units is given by the function g , where $r = g(q) = 40q$. Find $(g \circ f)(m)$. What does this composite function describe?

20. **Sociology** Studies have been conducted concerning the statistical relations among a person's status, education, and income. Figure 2.4 is a scatter plot of statistical data of status based on annual

$(g \circ f)(x)$.

10. If $f(x) = x^2 + 2x - 1$, find $(f \circ f)(x)$.

income I . For a certain population, suppose

$$S = f(I) = 0.45(I - 1000)^{0.53}$$

⁴R. K. Lusk and B. F. Moskon, *Mathematical Sociology* (Englewood Cliffs, NJ: Prentice Hall, 1975).