Panel 1

Final Topics: Financial Mathematics

Recall **Compound Interest Formula**: If you invest a principal P at an interest rate r per period compounded for n periods in total, you have:

$$S = P(1+r)^n$$

Ex: $1000 at 4% (nominal or APR) for 5 years compounded monthly.

$$S = 1000\left(1+\frac{0.04}{12}\right)^{5\cdot 12} = 1220.97$$

1

Panel 2

Ex: Suppose $500 is compounded semi-annually over 3 years and amounts to $588.38. What is the nominal interest rate?

$$S = P(1+r)^n$$

$$\Rightarrow 588.38 = 500(1+r)^6$$

$$\frac{588.38}{500} = (1+r)^6 \Rightarrow \sqrt[6]{\frac{588.38}{500}} = 1+r$$

$$r = 0.0275 \quad \text{semi-annual}$$

$$\Rightarrow R = 2\cdot r = 0.055 \text{ or } 5.5\%$$

2

Panel 3

Ex: How long will it take for $600 to amount to $900 at APR of 6% compounded quarterly?

$$900 = 600\left(1+\frac{0.06}{4}\right)^n$$

$$1.5 = \left(1+\frac{0.06}{4}\right)^n$$

$$1.5 = (1.015)^n \quad / \cdot \ln \text{ or } \log$$

$$\ln(1.5) = \ln\left[(1.015)^n\right] = n \cdot \ln(1.015)$$

$$\frac{\log \ln(1.5)}{\log \ln(1.015)} = n = 27.23 \Rightarrow \#\text{ years is } \frac{27.23}{4} = 6.8 \text{ years}$$

or 7 years

3

Panel 4

If you invest $P at 10% compounded quarterly for one year, it will earn more than 10% that year.

$$S - P = P\left(1+\frac{0.1}{4}\right)^4 - P = P\left(\left(1+\frac{0.1}{4}\right)^4 - 1\right) = P\left(1.025^4 - 1\right) =$$

$$P \cdot 0.1038$$

i.e. 10.38% of P

Def: The effective rate r_e equivalent to a nominal rate r compounded n times per year is:

$$r_e = \left(1+\frac{r}{n}\right)^n - 1$$

I.e. 10% compounded quarterly is effective rate of 10.38%

4

Panel 5

Effective rate $r_e = \left(1 + r/n\right)^n - 1$

Ex: Find effective rate equivalent to 6% compounded (a) semi-annually, (b) monthly

a) $r_e = \left(1 + \frac{0.06}{2}\right)^2 - 1 = 0.0609 \Rightarrow$ 6.09%

b) $r_e = \left(1 + \frac{0.06}{12}\right)^{12} - 1 = 0.06167 \Rightarrow$ 6.17%

5

Panel 6

Bank A: 6% cpd. daily

Bank B: 6.125% cpd semi-annually

Bank A: $r_e = \left(1 + \frac{0.06}{365}\right)^{365} - 1 = 0.06183$ or 6.18%

Bank B: $r_e = \left(1 + \frac{0.06125}{2}\right)^2 - 1 = 0.0621$ or 6.22%

Bank B wins!!!

6

Panel 7

Effective rate of 5% over ~~10~~ years, monthly cmpd.

$r_e = \left(1 + \frac{0.05}{12}\right)^{12} - 1 = 0.05116$ or 5.12% = 5.116

Invest $2000 over 10 years

did way $S = 2000\left(1 + \frac{0.05}{12}\right)^{120} = \$3294$

// about equal !

use r_e: $S = 2000(1 + 0.05116)^{10} = \$3295$

7

Panel 8

Present Value

8

Panel 9

Ex: Suppose you need $1,000 in three years, and your bank offers 9% compounded monthly. Find the present value of $1,000 in 3 years.

$$1000 = P\left(1+\frac{0.09}{12}\right)^{36}$$

$$1000\left(1+\frac{0.09}{12}\right)^{-36} = P \quad = \$764.15$$

$$\frac{1000}{\left(1+\frac{0.09}{12}\right)^{36}}$$

Formula that can handle more complicated situations

9

Panel 10

Present Value	*The PV function returns the present value of an investment. The present value is the total amount that a series of future payments is worth now. For example, when you borrow money, the loan amount is the present value to the lender.*
Excel Formula:	**=PV(rate, nper, pmt, fv)**, where
	rate: is the interest rate per period.
	nper: is the total number of payment periods in an annuity.
	pmt: is the payment made each period; it cannot change over the life of the annuity.
	fv: is the future value, or a cash balance you want to attain after the last payment is made.

10

Panel 11

Example:	*Find the present value of $1000 due after 3 years if the interest rate is 9% compounded monthly.*		
Solution:			
	Interest rate:	*0.09*	
	Compound Periods:	*12*	
	Number of years:	*3*	
	rate:	**0.0075**	= *Interest rate / compound periods*
	nper:	**36**	= *Number of years * compound periods*
	pmt:	**0**	
	fv:	**1000**	
	PV(rate, nper, pmt, fv) =	**($764.15)**	

11

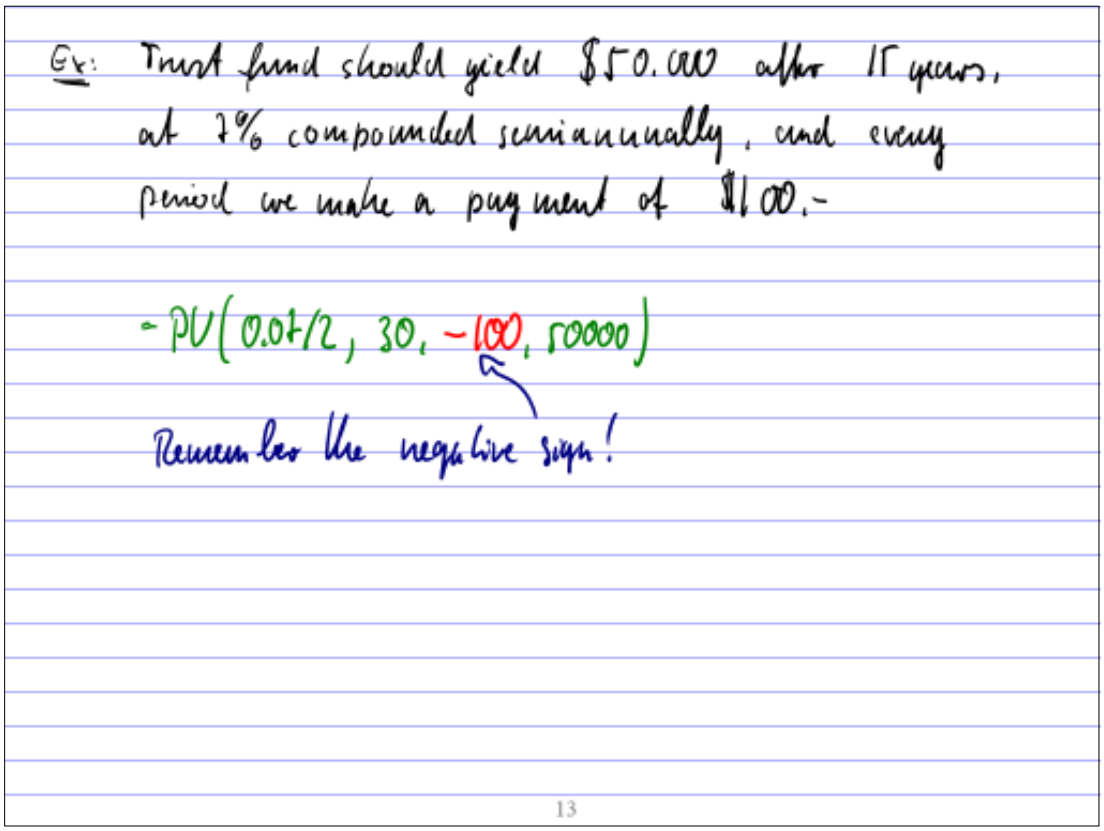
Panel 12

Ex: A trust fund for a child should yield $50,000 in 15 years at 7% comp. semiannually. How much should you invest?

= PV(0.07/2, 30, 0, 50000) = 17,813.92

12

Panel 13


Ex: Trust fund should yield $50.000 after 15 years, at 7% compounded semiannually, and every period we make a payment of $100.-
=PV(0.07/2, 30, -100, 50000)
Remember the negative sign!
13