Panel 1

Last Time

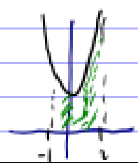
$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

represents graphically area under $f(x)$ as long as $f(x) \geq 0$

$$\int_a^b f(x) - g(x)\,dx$$ represents area between $f(x)$ and $g(x)$

as long as $f(x) - g(x) \geq 0$

Ex: Area under $f(x) = 3x^2 + 1$ from $x = -1$ to 2

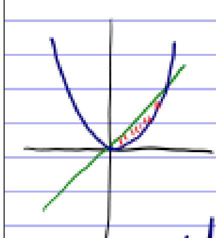


$$A = \int_{-1}^{2} 3x^2 + 1\,dx = x^3 + x\Big|_{-1}^{2} = (8+2) - (-1-1) = 12$$

1

Panel 2

Ex: Area between $f(x) = x^2$ and $g(x) = x$



$$\int x^2 - x\,dx \quad (A) = -1/6$$

$$\int x - x^2\,dx \quad (B)$$

Maple

set $x^2 = x$ to find bounds:

$x^2 - x = 0$

$x(x-1) = 0 \quad x = 0, x = 1$

int(x-x^2, x=0..1).

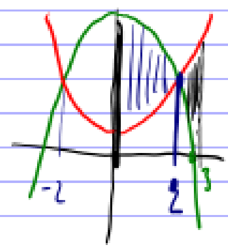
Area $$\int_0^1 x - x^2\,dx = \frac{1}{2}x^2 - \frac{1}{3}x^3\Big|_0^1 = \left(\frac{1}{2} - \frac{1}{3}\right) - (0-0) = \frac{1}{6}$$

2

Panel 3

Ex: Area between $9-x^2$ and x^2+1 from $x=0$ to $x=3$

$$\int_0^3 (9-x^2)-(x^2+1)\,dx = \text{attempt } 1$$



$$9-x^2 = x^2+1$$

$$0 = 2x^2-8$$

$$0 = 2(x^2-4) = 2(x+2)(x-2) \quad x = 2, -2$$

$$\int_0^2 \underbrace{(9-x^2)-(x^2+1)}\,dx = \int_0^2 8-2x^2\,dx = 8x-\frac{2}{3}x^3\Big|_0^2 = \#$$

add +

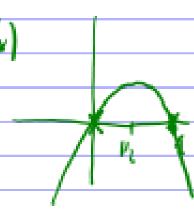
$$\int_2^3 \underbrace{(x^2+1)-(9-x^2)}\,dx = \int_2^3 2x^2-8\,dx = \frac{2}{3}x^3-8x\Big|_2^3 = \#$$

3

Panel 4

Area under $f(x) = x-x^2$ from $x=0$ to 1

(1) Draw $f(x)$

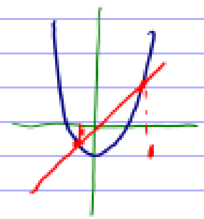


$$\int_0^1 x-x^2\,dx = \#$$

Area between $f(x) = x^2-1$ and $g(x) = x$ from

(1) Draw f, g in one system

$$\int_{\frac{1-\sqrt{5}}{2}}^{\frac{1+\sqrt{5}}{2}} x-(x^2-1)\,dx =$$



$$x^2-1 = x \Rightarrow x^2-x-1 = 0$$

$$x_1 = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}$$

4

Panel 5

Review: marginal profit is $P'(q) = 2q^2 - 1$ and profit for $q = 0$ is 8500. Find profit at $q = 10$!

$q = 1$

$$P(q) = \int 2q^2 - 1 \, dq = \frac{2}{3}q^3 - q + C$$

$$P(q) = \frac{2}{3}q^3 - q + C \quad \text{Also}$$

$$P(0) = \quad C = 500 \Rightarrow C = 500$$

$$P(1) = \frac{2}{3} - 1 + C = 500$$

$$P(q) = \frac{2}{3}q^3 - q + 500 \Rightarrow P(10) = \frac{2}{3}1000 - 10 + 500$$

5

Panel 6

Quiz 10

(1) Evaluate the following definite integrals

a) $$\int_0^1 x^2 \, dx = \frac{1}{3}x^3 \Big|_0^1 = \frac{1}{3}1^3 - \frac{1}{3}0^3 = \frac{1}{3}$$

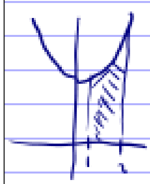
b) $$\int_1^4 4x - \sqrt{x} \, dx$$

6

Panel 7

(3) Suppose the marginal cost is $c'(q) = 0.3q^2 + 2q$ and the fixed cost is \$1000. Find the total cost at a production level of $q = 2$.

(4) Find the area under $f(x) = x^2 + 2$ from $x = 1$ to $x = 2$



$$\int_1^2 x^2 + 2\,dx = \frac{1}{3}x^3 + 2x \Big|_1^2 = \boxed{\left(\frac{8}{3} + 4\right) - \left(\frac{1}{3} + 2\right)}$$

$$= \frac{7}{3} + 2 = \frac{13}{3}$$

7

Panel 8

Final Topics: Financial Mathematics

Recall Compound Interest Formula: If you invest a principal P at an interest rate r per period compounded for n periods in total, you have:

$$S = P(1+r)^n$$

Ex: \$1000 at APR of 8% comp. quarterly for 5 years:

$$S = 1000 \cdot \left(1 + \frac{0.08}{4}\right)^{20} = \$1485.95$$

8

Panel 9

Ex: Suppose \$500 is compounded semi-annually over 3 years and amounts to \$588.38. What is the nominal interest rate?

$S = P(1+r)^n$

$588.39 = 500(1+r)^6$

$\frac{588.39}{500} = (1+r)^6$

$\sqrt[6]{\frac{588.39}{500}} - 1 = r = 0.0275$

$APR = 2 \cdot 0.0275 = 5.5\%$

9

Panel 10

Ex: How long will it take for \$600 to amount to \$900 at APR of 6% compounded quarterly?

$900 = 600\left(1 + \frac{0.06}{4}\right)^{4t}$

$\frac{900}{600} = \left(1 + \frac{0.06}{4}\right)^{4t}$ | $\ln(\ )$

Rest later

10