Panel 1

$$\int f(x)\,dx = \text{antiderivative } F(x) + C \quad (\text{indefinite integral})$$

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a) \quad (\text{definite integral})$$

Ex: $$\int 3x^4 - \sqrt{x} + \frac{5}{x}\,dx = 3\,\frac{1}{5}x^5 - \frac{2}{3}x^{3/2} + 5\ln(x) + C$$

$$\int_0^1 x^3 - x^2\,dx = \frac{1}{4}x^4 - \frac{1}{3}x^3\Big|_0^1 = \left(\frac{1}{4} - \frac{1}{3}\right) - (0 - 0) = -\frac{1}{12}$$

Qz on Monday

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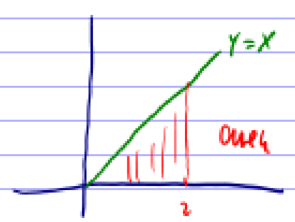
Panel 2

Meaning of Definite Integral

Thm: $\int_a^b f(x)\,dx$ gives you the area under the curve $f(x)$ as x goes from a to b, as long as $f(x) \ge 0$.

def. integral =

Ex: $$\int_0^2 x\,dx = \frac{1}{2}x^2\Big|_0^2 = \frac{1}{2}4 - \frac{1}{2}0 = 2$$



area $\frac{1}{2}bh = \frac{1}{2}\cdot 2\cdot 2 = 2$

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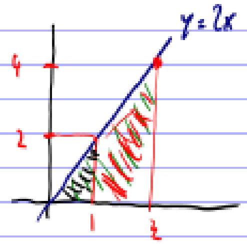
Panel 3

$$\int_1^2 2x\,dx = x^2\Big|_1^2 = 4-1 = 3$$

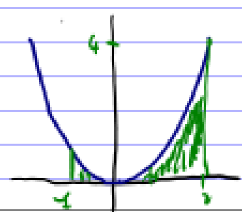


green area = $\frac{1}{2}\,2\cdot 4 = 4$

black area = $\frac{1}{2}\,1\cdot 2 = 1$

$\Rightarrow$ red area = $4-1 = 3$

Ex: Find area under $f(x) = x^2$ as $\boxed{x \text{ goes from } -1 \text{ to } 2}$



$$\int_{-1}^2 x^2\,dx = \frac{1}{3}x^3\Big|_{-1}^2 = \frac{1}{3}2^3 - \frac{1}{3}(-1)^3 = \frac{8}{3} + \frac{1}{3} = \frac{9}{3} = 3$$

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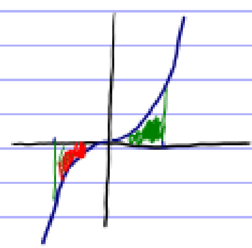
Panel 4

Find area under $f(x) = x^3$ from -1 to 1.

$$\int_{-1}^1 x^3\,dx = \frac{1}{4}x^4\Big|_{-1}^1 = \frac{1}{4} - \frac{1}{4} = 0$$

Note: $f(x)$ is not positive here!!



negative portion: $\int_{-1}^0 x^3\,dx = \frac{1}{4}x^4\Big|_{-1}^0 = 0 - \frac{1}{4} = -1/4$

positive portion: $\int_0^1 x^3\,dx = \frac{1}{4}x^4\Big|_0^1 = \frac{1}{4} - 0 = \frac{1}{4}$

total area: $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

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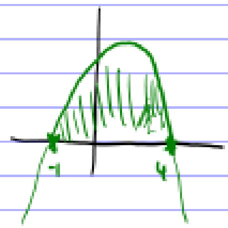
Panel 5

Find area under $f(x) = -x^2 + 3x + 4$



$$-x^2 + 3x + 4 = -(x^2 - 3x - 4) = -(x-4)(x+1)$$

$$\int_{-1}^{4} -x^2 + 3x + 4 \, dx = -\frac{1}{3}x^3 + \frac{3}{2}x^2 + 4x \Big|_{-1}^{4} =$$

$$= \left(-\frac{1}{3}4^3 + \frac{3}{2}4^2 + 4 \cdot 4\right) - \left(\frac{1}{3} - \frac{3}{2} - 4\right) = \#$$

pos.!!!

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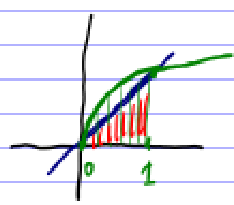
Panel 6

Area between curves

Ex: Area between $y = x$ and $y = \sqrt{x}$. $x = y^2$



green : $\int_0^1 \sqrt{x} \, dx$

red : $\int_0^1 x \, dx$

Area between: $\int_0^1 \sqrt{x} \, dx - \int_0^1 x \, dx = \int_0^1 \sqrt{x} - x \, dx =$

$$= \frac{2}{3}x^{3/2} - \frac{1}{2}x^2 \Big|_0^1 = \left(\frac{2}{3} - \frac{1}{2}\right) - (0 - 0) =$$

$$= \frac{2}{3} - \frac{1}{2} = \frac{4}{6} - \frac{3}{6} = \frac{1}{6}$$

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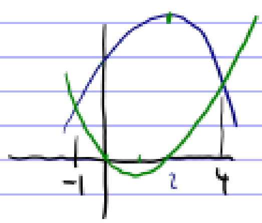
Panel 7

Thm: Area between $f(x)$ and $g(x)$ as x goes from a to b is given by $\int_a^b f(x)-g(x)\,dx$, as long as $f(x) \geq g(x)$

Ex: Area between $y = 4x-x^2+8$ and $y = x^2-2x$



area: $\int_{-1}^{4} (4x-x^2+8)-(x^2-2x) \; dx =$

$\int_{-1}^{4} 6x-2x^2+8\,dx = 3x^2-\frac{2}{3}x^3+8x\Big|_{-1}^{4} = \#$

↑ positive

Sound: $4x-x^2+8 = x^2-2x$

$0 = 2x^2-6x-8 = 2(x^2-3x-4) = 2(x-4)(x+1)$

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Panel 8

Reviewed some Maple commands:

int(f(x), x) - indef. integral

int(f(x), x=a..b) - def. integral $\int_a^b f(x)\,dx$

Also found critical pts, derivatives, etc.

See Maple 2 posted!

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