Panel 1

Anti-Derivatives

$\int f(x)\,dx$ gives a function whose derivative is $f(x)$

integral of f dx

Note: $\int f(x)\,dx = F(x) + C$, C is a constant

Ex: $\int 3x^2\,dx = 3\,\frac{1}{3}x^3 + C = x^3 + C$

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Panel 2

Rules of Integration

antipower rule: $\int x^p\,dx = \frac{1}{p+1}x^{p+1} + C$, $p \neq -1$

anti log rule $\int \frac{1}{x}\,dx = \ln(x) + c$

anti exp rule $\int e^x\,dx = e^x + c$

sums / differences

$$\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

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Panel 3

Examples:

$$\int \frac{1}{x^7}\,dx = \int x^{-7}\,dx = \frac{1}{-6}x^{-6} + C = -\frac{1}{6x^6} + C$$

$$\int 5x^2\,dx = 5\,\frac{1}{3}x^3 + C = \frac{5}{3}x^3 + C$$

$$\int 5\sqrt{x} - \frac{3}{x}\,dx = \int 5x^{1/2} - \frac{3}{x}\,dx = 5\frac{1}{\frac{3}{2}}x^{3/2} - 3\ln|x| + C =$$

$$= 5\,\frac{2}{3}x^{3/2} - 3\ln|x| + C =$$

$$\frac{10}{3}x^{3/2} - 3\ln(x) + C$$

$$\int \frac{5}{3x} + \frac{7}{2\sqrt{x}}\,dx = \int \frac{5}{3}\cdot\frac{1}{x} + \frac{7}{2}x^{-1/2}\,dx = \frac{5}{3}\ln|x| + \frac{7}{\cancel{2}}\cdot\cancel{2}\,x^{1/2} + C$$

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Panel 4

a) $\int \cancel{\ln x\,dx}$

b) $\int \cancel{e^x\,dx}$

c) $\int \cancel{5e^x + \frac{1}{3x}\,dx}$

$$\int 5e^x + \frac{1}{3x} + 7\sqrt{x^5} - 9\,dx$$

$$\sqrt{x^5} = x^{5/2}$$

$$5e^x + \frac{1}{3}\ln|x| + 7\,\frac{2}{5}x^{7/2} - 9x + C$$

$$\int 5e\,dx = 5e\cdot x + C$$

$$\int 5xe\,dx = \int 5e\,x\,dx = 5e\cdot\frac{1}{2}x^2 + C = \frac{5e}{2}x^2 + C$$

$$\int 5e^x\,dx = 5e^x + C$$

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Panel 5

If the fixed costs for producing Acme Quality Widgets is \$2000 and the marginal cost is

$C'(q) = 0.08q^2 - 1.6q + 6.5$, find the cost for producing 25 units.

$$C(q) = \int C'(q)\,dq = \int \frac{dC}{dq} \cdot dq = C$$

$$= \int 0.08q^2 - 1.6q + 6.5\,dq = 0.08 \cdot \frac{1}{3}q^3 - 1.6 \cdot \frac{1}{2}q^2 + 6.5q + D$$

$$C(q) = \frac{0.08}{3}q^3 - \frac{1.6}{2}q^2 + 6.5q + D \quad , \; C(0) = 2000$$

$$D = 2000$$

$$\Rightarrow C(q) = \frac{0.08}{3}q^3 - \frac{1.6}{2}q^2 + 6.5q + 2000 \quad \Rightarrow C(25)$$

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Panel 6

Marginal revenue $r'(q) = 2q + 5$ what's the demand.

Recall: $R(q) = p(q) \cdot q$

$$R(q) = \int 2q + 5\,dq = q^2 + 5q + C$$

$$R(0) = 0 = C$$

$R(q) = q^2 + 5q = q(q+5) \Rightarrow p(q) = q + 5$ is demand

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Panel 7

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Panel 8

Name: ______

Quiz #8

Evaluate the following indefinite integrals:

a) $\int 2x \, dx$

b) $\int x^4 \, dx$

c) $\int \sqrt[3]{x} \, dx$

d) $\int \frac{1}{x^4} \, dx$

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Panel 9

c) $\int e^x + \frac{1}{x}\,dx$

f) $\int 5x^2 - 10x^4 + 4\,dx$

g) $\int \frac{3}{2x^2} - \frac{7}{3}\sqrt{x}\,dx$

h) $\int (x+2)(3x-1)\,dx$

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Panel 10

If the fixed costs for producing Acme Quality Widgets is \$2000 and the marginal cost is $c'(q) = 0.08q^2 - 1.6q + 6.5$. Find the cost for producing 25 units.

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Panel 11

Definite Integral

$\int_a^b f(x)\,dx$ = integral of f(x) from a to b

Ex: $\int_0^1 x^2\,dx$ is definite integral of x^2 from 0 to 1

Thm: Fundamental Theorem of Calculus

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

where F(x) is antiderivative of f(x)

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Panel 12

$\int x^2\,dx = \frac{1}{3}x^3 + C$ indefinite integral - function

$$\int_0^1 x^2\,dx = \frac{1}{3}x^3\Big|_0^1 = \frac{1}{3}(1)^3 - \frac{1}{3}(0)^3 = 1/3$$ definite integral - number

$$= \frac{1}{3}x^3 + C\Big|_0^1 = \left(\frac{1}{3}(1)^3 + C\right) - \left(\frac{1}{3}0^3 + C\right) = \frac{1}{3}$$ (constant cancels)

Ex: $$\int_1^4 3x^4 - 7x^2\,dx = 3\frac{1}{5}x^5 - 7\frac{1}{3}x^3\Big|_1^4 = \left(\frac{3}{5}4^5 - \frac{7}{3}4^3\right) - \left(\frac{3}{5}1^5 - \frac{7}{3}1^3\right)$$

Meaning of definite integral on Monday!

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