Panel 1

Had fun with Derivatives ....


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Panel 2

## Anti-Derivatives

Derivative: $f(x) = \ldots \rightarrow f'(x) = \begin{cases} \text{const. rule} \\ \text{power rule} \\ \text{sum rule} \end{cases}$

Want to do the opposite: I give you a function $f$, you find a function $F(x)$ s.t. $F'(x) = f(x)$

$F$ is called antiderivative, or indefinite integral and is written as

integral of f dx

$$\int f(x)dx$$

↑ integrant

Ex: $\int 2x \, dx = x^2 + \text{constant}$ means a function whose derivative is $2x$

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Panel 3

Ex:

$\int 1\, dx = x + C$

Anti- Power Rule

$\int x\, dx = \frac{1}{2}x^2 + C$

$\int x^p\, dx = \frac{1}{p+1}x^{p+1} + C$

$\int x^5\, dx = \frac{1}{6}x^6 + C$

$p \neq -1$

$\int e^x\, dx = e^x + C$

$\int \sqrt{x}\, dx = \int x^{1/2}\, dx = \frac{2}{3}x^{3/2} + C$

$\int \frac{1}{x^3}\, dx = \int x^{-3}\, dx = -\frac{1}{2}x^{-2} + C$

$\frac{d}{dx}$

$\int \frac{1}{x}\, dx = \int x^{-1} dx = \ln|x| + C$

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Panel 4

More:

$\int x^2 + 2x\, dx = \frac{1}{3}x^3 + 2\frac{1}{2}x^2 + C = \frac{1}{3}x^3 + x^2 + C$

$\int 3x\, dx = 3 \cdot \frac{1}{2}x^2 + C = \frac{3}{2}x^2$

$\int 2\sqrt[5]{x^4} - 7x^3 + 10e^x - 1\, dx$

$\int 2x^{4/5} - 7x^3 + 10e^x - 1\, dx = 2 \cdot \frac{5}{9}x^{9/5} - 7\frac{1}{4}x^4 + 10e^x - 1x + C$

$\int 5x^2 + 7x + 10x^4 + 7x^6\, dx =$

$5\frac{1}{3}x^3 + 7\frac{1}{2}x^2 + 10\frac{1}{5}x^5 + 7\frac{1}{7}x^7 =$

$\frac{5}{3}x^3 + \frac{7}{2}x^2 + 2x^5 + x^7 + C$

$\boxed{\text{Quiz Wed}}$

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Panel 5

$\int 2x^3\,dx = 2\,\frac{1}{4}x^4 + C$

$\int e^x + 7\,dx = e^x + 7x + C$

$\int 4\sqrt{x^3}\,dx = \int 4\,x^{3/2}\,dx = 4\,\frac{2}{5}x^{5/2} + C$

$\int x^5 + 9\,dx = \frac{1}{6}x^6 + 9x + C$

$\int \frac{1}{2}\cdot e^x\,dx = \frac{1}{2}e^x + C$

$\int 3x^7 + \frac{7}{2}\sqrt{x}\,dx$

$3\,\frac{1}{8}x^8 + \frac{7}{2}\,\frac{2}{3}x^{3/2} + C$

$\int \frac{7}{x}\,dx = \int 7\cdot\frac{1}{x}\,dx = 7\,\ln|x|$

$\int \frac{5}{3x^2}\,dx = \int \frac{5}{3}x^{-2}\,dx = \frac{5}{3}\cdot\frac{1}{-1}x^{-1} = -\frac{5}{3}x^{-1} + C$

$\int (x+2)^2\,dx = \int x^2 + 4x + 4\,dx$

$= \frac{1}{3}x^3 + x^2 + 4x + C$

$\int e^{\pi}\,dx = e^{\pi}x$, $\int 7\,dx = 7\cdot x$

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Panel 6

Even more:

① Find a function $y$ such that $y' = 8x - 4$ and $y(2) = 5$

initial condition

Answer. $\int 8x - 4\,dx = 8\,\frac{1}{2}x^2 - 4x + C$

$y(x) = 4x^2 - 4x + C$

$y(2) = 4\cdot 4 - 4\cdot 2 + C = 8 + C = 5$

$\Rightarrow C = -3$

$y(x) = 4x^2 - 4x - 3$

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Panel 7

(i) Find a function $y$ such that $y'' = x^2 - 6$ and also $y'(0) = 2$ and $y(1) = -1$

If $y'' = x^2 - 6$

$y' = \frac{1}{3}x^3 - 6x + C$ , $y'(0) = C = 2$

$y' = \frac{1}{3}x^3 - 6x + 2$

$y = \frac{1}{3}\frac{1}{4}x^4 - 6\frac{1}{2}x^2 + 2x + D$

$= \frac{1}{12}x^4 - 3x^2 + 2x + D$

$y(1) = \frac{1}{12} - 3 + 2 + D = -1 \Rightarrow D = \_\_\_\_$ ?

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Panel 8

Ex: Suppose the marginal revenue function for a product is $\frac{dr}{dq} = 2000 - 20q - 3q^2$

Find the demand function

$R(x) = \int 2000 - 20q - 3q^2 dq = 2000q - 20\frac{1}{2}q^2 - 3\frac{1}{3}q^3 + C$

$= \int \frac{dR}{dq} dq$

Recall $R = q \cdot p(q)$

$= 2000q - 10q^2 - q^3 + C$

$R(q) = 2000q - 10q^2 - q^3 + C$

Know $R(0) = 0$.

$R(0) = C = 0$

$R(q) = 2000q - 10q^2 - q^3 = q[2000 - 10q - q^2] \Rightarrow \boxed{p(q) = 2000 - 10q - q^2}$

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Panel 9

Rules of Integration

$$\int x^p\,dx = \frac{1}{p+1}x^{p+1} + C \quad \text{for } p \neq -1$$

$$\int \frac{1}{x}\,dx = \ln(x) + C$$

$$\int e^x\,dx = e^x + C$$

sums / differences go individually!

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Panel 10

e) $\int \frac{2x^2}{7} - \frac{8}{3}x^4\,dx = \frac{2}{7}\cdot\frac{1}{3}x^3 - \frac{8}{3}\frac{1}{5}x^5 + C$

f) $\int (x^2+5)(x-3)\,dx$ do it

g) $\int \frac{2}{\sqrt{x^3}} - \frac{\sqrt[4]{x^5}}{8}\,dx = \int 2\cdot x^{-3/2} - \frac{1}{8}x^{5/4}\,dx =$

$= 2(-2)x^{-1/2} - \frac{1}{8}\frac{4}{9}x^{9/4} + C$

h) $\int (3x+2)^3\,dx$

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