Panel 1

Limits: $\lim_{x \to a} f(x) = L$, one-sided limits, limits at infinity

by using a table

Continuity: (1) $f(c)$ exists

(2) $\lim_{x \to c} f(x)$ exists

(3) (1) = (2)

f is cont. at c: if there is no hole or gap in graph

Derivative: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ slope of tangent, inst. rate of change,

rules for diff marginal ..., velocity

higher order derivatives

local extrema, incr., decr., concavity, inflection points

1

Panel 2

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x \le 2 \\ kx & \text{if } x > 2 \end{cases}$$

cont. at $x = 2$? for which k?

a) $f(2) = \underline{3}$

$\lim_{x \to 2^+} kx = 2k$

b) $\lim_{x \to 2} f(x) = \underline{3}$

$\lim_{x \to 2^-} x^2 - 1 = 3$

Need $2k = 3$

$\boxed{k = \frac{3}{2}}$

c) (1) = (2)

2

Panel 3

$$f(x) = x^2 - 6x + 3$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{\left((x+h)^2 - 6(x+h) + 3\right) - \left(x^2 - 6x + 3\right)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{6x} - 6h + \cancel{3} - \cancel{x^2} + \cancel{6x} - \cancel{3}}{h} =$$

$$= \lim_{h \to 0} \frac{2xh + h^2 - 6h}{h} =$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x + h - 6)}{\cancel{h}} = \lim_{h \to 0} 2x + \cancel{h}^{0} - 6 =$$

$$= 2x - 6$$

$$\Rightarrow \text{at } x = 2: \; f'(2) = -2, \quad f(2) = -5 \qquad y + 5 = -2(x - 2)$$

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Panel 4

$$f(x) = \frac{4}{t} + \frac{t}{4} + \sqrt[5]{t^2} + 5e^x - \ln(5)$$

$$= 4 \cdot t^{-1} + \frac{1}{4} \cdot t + t^{2/5} + 5e^x - \ln(5)$$

$$f'(x) = 4(-1)t^{-2} + \frac{1}{4} + \frac{2}{5} t^{-3/5} + 5e^x - 0$$

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Panel 5

$f(x) = x^3 - 9x^2 + 15x - 4$

$f'(x) = 3x^2 - 18x + 15 = 0$

$3(x^2 - 6x + 5) = 0$

$3(x - 5)(x - 1) = 0$ $\quad x = 1, 5$ are critical

	A	C	B
	0 · 1	2 · 5	6
f'	+	−	+
f	↗	↘	↗

$x = 1$ is max

$x = 5$ is min

f increasing on A and B $(-\infty, 1) \cup (5, \infty)$

f decreasing on C $\quad (1, 5)$

5

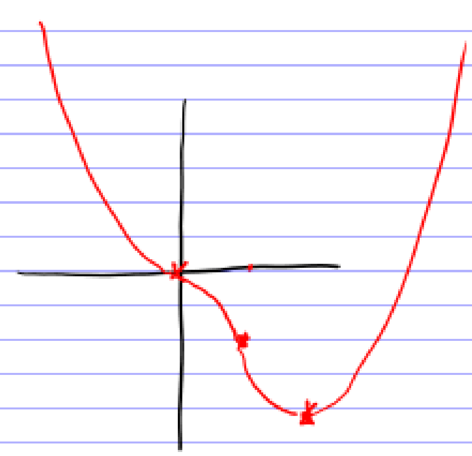
Panel 6

$f(x) = 2x^4 - 4x^3$

$f'(x) = 8x^3 - 12x^2 = 4x^2(2x - 3) = 0 \Rightarrow x = 0, \frac{3}{2}$ critical

$f''(x) = 24x^2 - 24x = 24x(x - 1) = 0 \Rightarrow x = 0, 1$ possible infl. points

		0	1	3/2
f'	−	−	−	+
f''	+	−	+	+
f	↘	↘	↘	↗



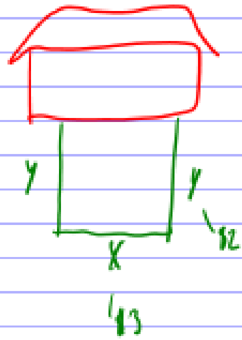
$f(0) = 0$

$f(1) = -2$

$f(3/2) = -?4.7$

6

Panel 7



Know: area $= 10\,800 = xy \Rightarrow \underline{y = \frac{10\,800}{x}}$

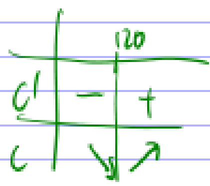
$C = 3x + 2y \cdot 2 = 3x + 4y$ is $\underline{\text{cost of fence}}$

$\Rightarrow C(x) = 3x + 4 \cdot \frac{10800}{x} = 3x + \frac{43200}{x}$

$C'(x) = 3 - \frac{43200}{x^2} = 0$

$3 = \frac{43200}{x^2}$

$x^2 = \frac{43200}{3} = 14400$



$\underline{x = \pm 120} = \underline{120}$ is $\underline{\text{min}}$

$\underline{y} = \frac{10800}{120} = \underline{90}$

7

Panel 8

Cost function $C(q)$

Avg. cost: $\bar{C}(q) = \frac{C(q)}{q}$ $\quad q\bar{C}(q) = C(q)$

marginal cost: $C'(q)$

fixed cost : $C(0)$

demand : $p(q) = \ldots$

Revenue: $R(q) = q \cdot p(q)$

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