Panel 1

Differentiation Recipes

First Derivative: $f' > 0$: f is increasing $\nearrow$

$f' < 0$: f is decreasing $\searrow$

$f(x) = x^3$ $x=0$ is critical but not min/max

$f' = 0$: critical points

or f' d.n.e. potential max/mins

Second Derivative: $f'' > 0$: f is concave up

$f'' < 0$: f concave down

$f'' = 0$: possible infl. points

or f'' d.n.e.

curvature could change

1

Panel 2

Recipe for Local Extrema: (max/min)

① Find $f'(x)$ first deriv.

② Solve $f'(x) = 0$ critical points

(and f' d.n.e.)

③ Draw table with signs of f' determine f $\nearrow$ or $\searrow$

signs of f'

④ Provide answer

2

Panel 3

Recipe for Concavity

1. Find $f''(x)$ — second deriv.
2. Solve $f''(x) = 0$ (or f'' d.n.e.) — possible infl. points
3. Create table with signs of f'' — f'' signs tell you if f $\frown$ $\smile$
4. Provide answers

3

Panel 4

Recipe for Curve Sketching

1. Solve $f'(x) = 0$ or f' d.n.e. — critical pts
2. Solve $f''(x) = 0$ or f'' d.n.e. — possible infl. points
3. Draw table with signs of f' and f'' using all points from 1 and 2 — use all critical and possible infl. points
4. Find y-intercept and x-intercept(s) if possible — y-int: $f(0)$; x-int: $f(x) = 0$ ~ tricky
5. Evaluate f at 1 and 2 — to get the scale of f
6. Put everything together — label all points

4

Panel 5

Ex: Find local extrema for $f(x) = \frac{1}{3}x^3 - 2x^2 + 3x + 1$

(1) $f'(x) = x^2 - 4x + 3$

(2) $0 = x^2 - 4x + 3 = (x-1)(x-3)$ $x = 1$, $x = 3$ are critical

(3)

	0	1	2	3	1000000
f'	+		−		+
f	↗		↘		↗

$x=1$ is max

$x=3$ is min

5

Panel 6

Ex: Find concavity for $f(x) = x^4 + 2x^3 - 12x^2$

(1) $f'(x) = 4x^3 + 6x^2 - 24x$

$f''(x) = 12x^2 + 12x - 24$

(2) $0 = 12(x^2 + x - 2) = 12(x+2)(x-1)$, $x = -2$, $x = 1$ possible infl. points

(3)

	-3	-2	0	1	2
f''	+		−		+
f	$\cup$		$\cap$		$\cup$
	I		II		III

$x=-2$ and $x=1$ are indeed inflection points

and f is concave up $(-\infty,-2) \cup (1,\infty)$ I and III

and concave down in $(-2,1)$ II

6

Panel 7

Find concavity for $f(x) = x^4 - 4x$

$f'(x) = 4x^3 - 4$

$f''(x) = 12x^2 = 0 \Rightarrow x = 0$ is possible infl. point

		0	
f''	$+$		$+$
f	$\cup$		$\cup$

no inflection points, f is always concave up

7

Panel 8

Ex: Sketch the function $f(x) = x^4 - 2x^2$

① $f'(x) = 4x^3 - 4x = 0$

$4x(x^2-1) = 0 \Rightarrow x = 1, -1, 0$

② $f''(x) = 12x^2 - 4 = 0$

$12x^2 = 4$, $x^2 = \frac{4}{12} = \frac{1}{3} \Rightarrow x = \pm\sqrt{1/3}$

③

	-1	$-\sqrt{1/3}$	0	$\sqrt{1/3}$	1	
f'	$-$	$+$	$+$	$-$	$-$	$+$
f''	$+$	$+$	$-$	$-$	$+$	$+$
f						

④ $f(0) = 0$

⑤ $f(1) = -1$

$f(-1) = -1$

$f(+\sqrt{1/3}) = \text{calc.}$

$f(-\sqrt{1/3}) = \text{calc.}$

8

Panel 9

Ex: $f(x) = x^2 - 50 \cdot \ln(x)$, $x > 1$

a) Max / min

$f'(x) = 2x - \frac{50}{x} = 0$ $\quad 2x = \frac{50}{x}$ $\quad 2x^2 = 50$ $\quad x^2 = 25$ $\quad x = +5$ or

		5	
f'	$-$		$+$
f	$\searrow$		$\nearrow$

$x = 5$ is a min.

b) concavity

$f''(x) = 2 - 50(-x^{-2}) = 2 + \frac{50}{x^2} > 0$ $\quad$ no infl. points

f is concave up

9

Panel 10

Carefully sketch the graph of $y = 2x^3 + 3x^2 - 12x - 3$. Identify all critical points and points of inflection. State the intervals over which the graph is increasing, decreasing. Concave up and concave down. Identify any absolute or relative extrema. (8%).

HW

10

Panel 11

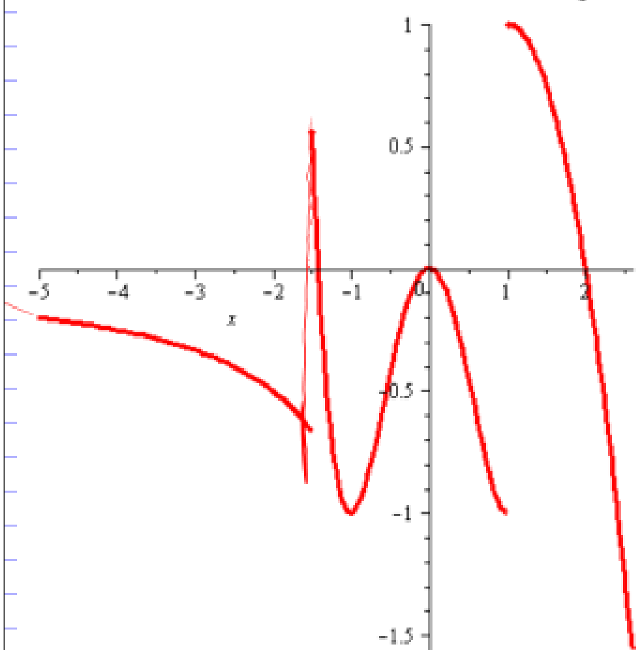
Ex: To provide security, a manufacturer plans to fence in a 10,800 ft^2 rectangular storage area adjacent to a building. The fence parallel to the building costs \$3 per foot, the other sides are \$2 per foot.

next line

11

Panel 12

Find the signs of the given quantities



$f(0)$	0	$f'(0)$	0
$f''(0)$	−	$f''(2)$	−
$f'(2)$	−	$f(2)$	0
$f'(-3)$	−	$f''(-3)$	−

$\lim_{x\to 1^+} f(x)$ 1 $\quad \lim_{x\to 1^-} f(x)$ −

$\lim_{x\to -\infty} f(x)$ 0 $\quad \lim_{x\to \infty} f(x) = -\infty$ d.n.e.

$\lim_{x\to -1.5} f(x)$ d.n.e.

12