Panel 1

Different Terms for "Derivative":

Derivative
- rates
- slope
- rate of change
- marginal cost

Special derivatives: $\frac{d}{dx}(e^x) = e^x$, $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$

Higher Order derivatives: f'' is rate of change of f'

If f is distance $\Rightarrow$ f' is velocity, f'' = accel.

Finding Max/Mins

later again

1

Panel 2

Examples: $C(x) = 5\ln(x) + 7e^x - x^4 + 2\sqrt{x} + \cancel{\log(\pi)}$

Find rate of change of marginal cost when $x = 1$

marginal cost: $C'(x) = 5 \cdot \frac{1}{x} + 7e^x - 4x^3 + 2\frac{1}{2}x^{-1/2} + 0$

$= \frac{5}{x} + 7e^x - 4x^3 + x^{-1/2}$

want. $C''(x) = -5x^{-2} + 7e^x - 12x^2 - \frac{1}{2}x^{-3/2}$

$\frac{5}{x} = 5 \cdot \frac{1}{x} = 5 \cdot x^{-1}$

$\Rightarrow \frac{d}{dx}\left(\frac{5}{x}\right) = 5(-1)x^{-2}$

$= -5x^{-2}$

if say, $C'(10) = 15$

II, $C''(10) = -7$

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Panel 3

$C(q) = 0.2q^2 + 2q + 500$

How fast is marg. cost changing when $q = 97.357$

C'' C'

$C'(q) = 0.4q + 2$

$C''(q) = 0.4$

$\rightarrow C''(97.357) = \underline{0.4}$

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Panel 4

Name: __________

Quiz #7

① If $P(x) = 10\ln(x) + e^x$ is a profit function

a) find the marginal profit for $x = 1$

b) Should you increase or decrease production from its current level of $x = 1$?

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Panel 5

(2) Find the indicated derivative for the function:

a) $f(x) = 3x^2 - 2\sqrt{x} + 3 \cdot \ln(x)$; find $f''(x)$

b) $f(x) = 4x^3 - 3x^2 + 7x - 9$; find $f^{(10)}(x)$, the 10^{th}-deriv

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Panel 6

How to Find local Max / Min

Min or max

Ex: $f(x) = 2x^3 + 3x^2 - 36x + 3$ Find all local extrema

(1) $f'(x)$

(2) $f'(x) = 0$

critical points

(3) Make a table of signs of f' using the critical points

(4) Answer

(1) $f'(x) = 6x^2 + 6x - 36$

(2) $6(x^2 + x - 6) = 0$

$6(x+3)(x-2) = 0 \Rightarrow x = -3, \; x = 2$

	$-\infty$ -3	0 2	3
f'	+	−	+
f	↗	↘	↗

max   min

$x = -3$ is max

$x = 2$ is min

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Panel 7

Ex: $f(x) = 3x^2 - 5x + 9$ Find local max / min

① $f'(x) = 6x - 5$

② $6x - 5 = 0$ $x = 5/6 \sim 1$

③

	0	5/6	10
f'	$-$		$+$
f	$\searrow$	min	$\nearrow$

$x = 5/6$ is a min.

(also called, in this case, the vertex $-b/2a$)

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Panel 8

Ex: Suppose $C(x) = \frac{360000}{x} + 4x$ is a cost function based on the inventory $x \geq 0$. How much inventory should you carry to minimize the cost?

① $\boxed{C'(x) = -360000\, x^{-2} + 4 = -\frac{360000}{x^2} + 4}$

$-360\,000\, x^{-2} + 4 = 0$

$4 = \frac{360000}{x^2}$

$x^2 = 90\,000$

$x = \pm 300$

$x = 300$ Thus, carry 300 items for min. cost

	1	300	
C'	$-$		$+$
C	$\searrow$		$\nearrow$

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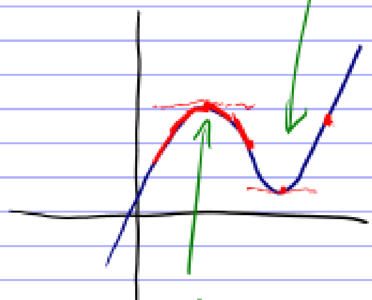
Panel 9

First vs Second Derivative

First deriv. tells you rate of increase or decrease of f

Thus, f'' tells you rate of incr. or decr. of f'

f'' is positive because f' is increasing



f'' tells you about the curvature of f

f'' is negative because f' is decreasing

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Panel 10

Meaning of f''

$f'' > 0 \Rightarrow f$ is concave up, i.e. $\cup$ (think x^2)

$f'' < 0 \Rightarrow f$ is concave down, i.e. $\cap$ (think $-x^2$)

$f'' = 0 \Rightarrow$ possible points of inflection, i.e. possible points where curvature changes.

Ex: $f(x) = 2x^3 - 6x^2$ Investigate concavity

$f'(x) = 6x^2 - 12x$

$f''(x) = 12x - 12 = 0 \Rightarrow x = 1$

		$x=1$
f''	$-$	$+$
f	$\cap$	$\cup$

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Panel 11

Recipe for Curve Sketching : $f(x) = x^3 - 9x^2 + 15x - 4$

① Find f'
Solve $f' = 0$

② Find f''
Solve $f'' = 0$

③

f'			
f''			
f			

④ Find $f(x)$ at special points.

① $f'(x) = 3x^2 - 18x + 15$ $x = 1, 5$ are critical

$3(x^2 - 6x + 5) = 3(x-5)(x-1) = 0$

② $f''(x) = 6x - 18 = 0 \Rightarrow x = 3$ possible inf. point

③

	0	1	2	3	4	5	1000
f'	+		−		−		+
f''	−		−		+		+
f							

$f(1) = 3$, $f(3) = -13$, $f(5) = -29$

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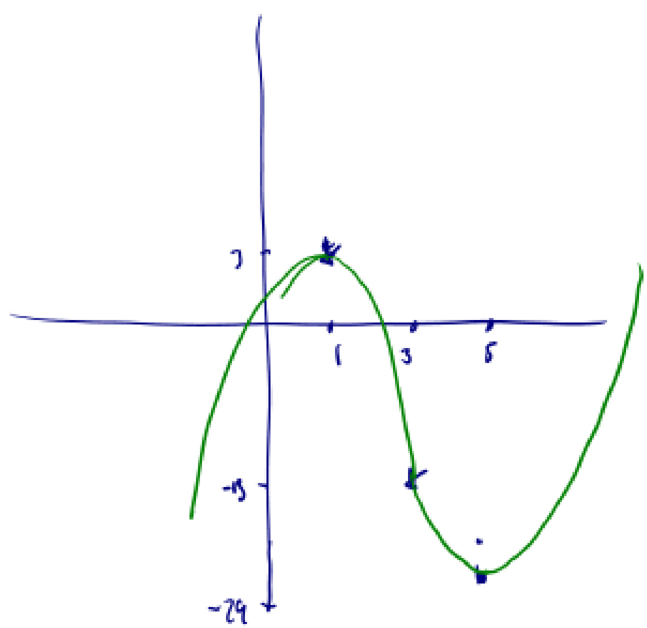
Panel 12



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