Panel 1

Different Terms for "Derivative":

Definition: $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$

deriv
- slope of tangent
- (inst.) rate of change
- velocity
- change in price
- marginal cost (revenue, profit) of producing one more unit
- tells us if f goes up or down

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Panel 2

Find marginal cost of

$\overline{c(q)} = 0.01q + 5 + \frac{500}{q}$ when $q = 15$

$\bar{c}$ is avg. cost function, i.e. $\bar{c} = c/q$

$\overline{c(q)} = 0.01q + 5 + \frac{500}{q}$

$c(q) = q \cdot \overline{c(q)} = 0.01q^2 + 5q + 500$

$c'(q) = 2 \cdot 0.01q + 5 = 0.02 \cdot q + 5$ at $q = 15$: $0.3 + 5 = \underline{5.3}$

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Panel 3

Derivatives of special functions

If $f(x) = e^x$ then $\boxed{f'(x) = e^x}$

$$f'(x) = \lim_{h\to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h\to 0} \frac{e^x e^h - e^x}{h} = \lim_{h\to 0} e^x \frac{(e^h - 1)}{h} =$$

$$= e^x \lim_{h\to 0} \frac{e^h - 1}{h} = e^x \cdot 1 = e^x$$

h	$\frac{e^h - 1}{h}$
0.01	1.005
0.001	1.0005
	$\downarrow$
	1

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Panel 4

Derivative of e^x and $\ln(x)$:

$$\frac{d}{dx} e^x = e^x \quad !$$

$$\frac{d}{dx} \ln(x) = \frac{1}{x} \quad !$$

Ex: $f(x) = e^2 + e^x + x^2 + x^e + \ln(x) + \frac{1}{x} + x$ ($\frac{1}{x} = x^{-1}$)

$$f'(x) = 0 + e^x + 2x + e x^{e-1} + \frac{1}{x} - x^{-2} + 1$$

Q6 on Monday!

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Panel 5

Ex: A cost function is given by

$$C(q) = 25\ln(q) + q^2 \quad , q \geq 1$$

Find the marginal cost when $q = 6$ and interprete it.

$$C'(q) = 25 \cdot \frac{1}{q} + 2q$$

$$C'(6) = \frac{25}{6} + 12 = \#$$

Means: if production changes from 6 to 7 units, cost goes up by #

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Panel 6

Higher Order Derivatives

If $f(x)$ is a (differentiable) function, then $f'(x)$ is again a function. Do it again:

$f''(x)$ is derivative of f' or 2^{nd} deriv. of f or $\frac{d^2}{dx^2}$

$f'''(x)$ is the deriv of f'' or 3^{rd} deriv. of f or $\frac{d^3}{dx^3}$

$\vdots$ $\vdots$

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Panel 7

Ex: If $f(x) = 3x^4 - 9x^2 + 5x$, find f''

$$f'(x) = 12x^3 - 18x + 5$$

$$f''(x) = 36x^2 - 18$$

Ex: $f(x) = 5e^x - 7\ln(x) + \sqrt{x}$ ($x^{1/2}$). Find f''

$$f'(x) = 5e^x - 7 \cdot \frac{1}{x} + \frac{1}{2}x^{-1/2}$$

$$f''(x) = 5e^x + 7x^{-2} - \frac{1}{4}x^{-3/2}$$

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Panel 8

Sometimes higher order derivatives become easy:

Ex: $f(x) = 3x^3 - 2x^2 + x$. Then

$$f'(x) = 6x^2 - 4x + 1$$

$$f''(x) = 12x - 4$$

$$f'''(x) = 12$$

$$\Rightarrow f''''(x) = f^{(4)}(x) = 0$$

$$f'''''(x) = f^{(5)}(x) = 0$$

$$\vdots$$

$$f^{(100)}(x) = 0$$

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Panel 9

Interpretation: f is a function

f' rate of change of f

f'' is rate of change of f'

f' is useful and has many interpretations, f'' is less useful. But:

If $s(t)$ is distance function, then

$s'(t)$ is velocity

$s''(t)$ is acceleration

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Panel 10

Ex: Find rate of change of $f''(x)$ if $f(x) = x + \ln(x)$

$f'(x) = 1 + \frac{1}{x}$

$f''(x) = -x^{-2}$

Rate of change of f'' is $f'''(x) = 2x^{-3}$

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Panel 11

Ex: Suppose $s(t) = -16t^2 + 10t + 5$ is the distance function for some object. Find

a) velocity and acceleration after 10 seconds

$s'(t) = -32t + 10$ velocity $s'(10) = -320 + 10 = \underline{\underline{-310}}$

$s''(t) = -32$ accel.

b) When is the velocity zero? Interprete!

$s'(t) = -32t + 10 = 0 \iff 32t = 10$ $\underline{\underline{t}} = \frac{10}{32} = \underline{\underline{0.315}}$

is the peak of the parabola!

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Panel 12

Ex: If $\bar{c}(q) = 0.2q + 2 + \frac{500}{q}$ is the average cost then find.

a) $C(q) = q \cdot \bar{c}(q) = \underline{0.2}q^2 + 2q + 500$

b) Fixed cost: $C(0) = 500$

c) Marginal cost: $c'(q) = 0.4q + 2$

d) Rate of change of marginal cost

$\underline{\underline{c''(q) = 0.4}}$

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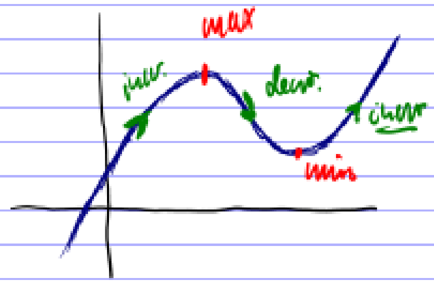
Panel 13

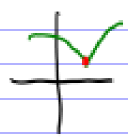
More appl. of derivative: Curve Sketching



f is increasing, i.e. f goes up

f is decreasing, i.e. f goes down

Theorem: If f has a (local) max or min. at $x=c$, then $f'(x)=0$ at $x=c$ or $f'(x)$ is not differentiable at $x=c$



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Panel 14

How to Find local Max / Min

Ex: $f(x) = 2x^3 + 3x^2 - 12x - 3$ Find all local extrema (max. or min.)

① $f'(x) = 6x^2 + 6x - 12$

② $0 = 6(x^2 + x - 2) = 6(x+2)(x-1)$

$x = -2, 1$

③

	-3	-2 (max)	0	1 (min)	2
f'	+		−		+
f	↗		↘		↗

① Find $f'(x)$

② $f'(x) = 0$ for x, called critical points

③ Make a table with signs of f', using "test points"

④ Draw f ↗ or ↘ and answer

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Panel 15

Ex: $f(x) = x^2 + 6x - 8$ Find local ~~max/min~~ extrema

$f'(x) = 2x+6$

$0 = 2x+6 \Rightarrow x = -3$ is the critical point

	$-\infty$	-3 min	∞
f'	$-$		$+$
f	$\searrow$		$\nearrow$

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Panel 16

Ex: Suppose $C(x) = \frac{360000}{x} + 4x$ is a cost function based on the inventory $x \geq 0$. How much inventory should you carry to minimize the cost?

HW

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