Panel 1

Differentiation Shortcuts $\frac{d}{dx}, \frac{df}{dx}, \frac{dy}{dx}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

f' is the slope of the tangent line

Power Rule: $\frac{d}{dx}[x^n] = nx^{n-1}$ Ex: $\frac{d}{dx} x^3 = 3x^2$, $\frac{d}{dx} x =$

Sum Rule: $\frac{d}{dx}(f(x) \pm g(x)) = f' \pm g'$

Constant Factor: $\frac{d}{dx}(c \cdot f(x)) = c \cdot f'$

Product Rule, Quotient Rule, Chain Rule ⇒ Maple

1

Panel 2

(1) $f(x) = x^8$ ✓ $f' = 8x^7$

(2) $f(x) = 3x^5 - 2x^4 + 7x + 2$ ✓ $f'(x) = 15x^4 - 8x^3 + 7$

(3) $f(x) = 4\sqrt[3]{x^2} - 4\sqrt[2]{x^3} = 4x^{\frac{2}{3}} - 4x^{\frac{3}{2}}$, $f'(x) = 4 \cdot \frac{2}{3} x^{2/3-1} - 4 \frac{3}{2} x^{3/2-1}$

(4) $f(x) = \pi x + ex^3$ $f' = \pi + e3x^2$

(5) $f(x) = \frac{1}{x^2} - x^3 = x^{-2} - x^3 \Rightarrow f'(x) = -2x^{-2-1} - 3x^2 = -2x^{-3} - 3x^2$

(6) $f(x) = \frac{5}{3} \frac{1}{\sqrt[3]{x^4}} + \frac{\pi^2}{\sqrt{e}} = \frac{5}{3} x^{-\frac{4}{3}} + \frac{\pi^2}{\sqrt{e}}$, $f'(x) = \frac{5}{3}\left(-\frac{4}{3}\right)x^{-4/3-1} + 0$

(7) $f(x) = x^2(3x^2 - 5x - 1)$ ✓

(8) slope of tangent to $y = 2x^3 - 3x$ when $x = 2$ / 4

2

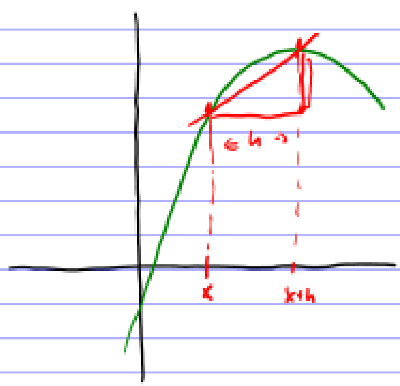
Panel 3

Ex: $f(x) = x^4(3x^2 - 2x + 1)$

3

Panel 4

Why study Derivatives?



$\frac{f(x+h) - f(x)}{h} = \frac{\text{change in } y}{\text{change in } x}$

= avg. rate of change

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ = instantaneous Rate of change $= f'$

4

Panel 5

Ex: Suppose the position function of an object is given by $f(t) = 3t^2 + 5$, t is time in sec

a) Find avg. rate of change over [2, 3]

$f(2) = 17$ m, $f(3) = 32$ m $\Rightarrow \frac{f(3) - f(2)}{3-2} = \frac{32-17}{1} = \frac{15}{1}$

i.e. avg. velocity is 15 m/sec

b) how fast is the car going when $t = 2$?

$f'(t) = 6t + 0 = 6t$, at $t = 2 \Rightarrow$ inst. rate of change is 12 m/sec

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Panel 6

Ex: Let $p = 100 - q^2$ be a demand function.
How fast is price changing when $q = 5$?

$p'(q)$ for $q = 5$

$p'(q) = -2q \Rightarrow p'(q=5) = -10$

If demand is currently at 5 units, an increase of 1 unit will result in \$10 decrease in price!

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Panel 7

Ex: A sociologist believes that x years after the beginning of a program, $f(x)$ - thousand preschoolers will be enrolled, where

$$f(x) = \frac{10}{9}\left(12x - x^2\right), \quad 0 \le x \le 12$$

Suppose the program is in existence for 9 years. How much increase in enrollment is to be expected?

$f'(x=9)$

$$f(x) = \frac{10}{9} \cdot 12x - \frac{10}{9}x^2 \Rightarrow f'(x) = \frac{10}{9} \cdot 12 - \frac{10}{9} \cdot 2x \Rightarrow f'(x=9) = \frac{120}{9} - 20 = -6.6$$

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Panel 8

Ex: Suppose revenue function is $R(q) = 9q - q^3$ and the current level of production is at $q = 2$ (thousand). Should you increase or decrease production?

Is the level of production increasing, i.e. rate of change positive?

$$R'(q) = 9 - 3q^2 \Rightarrow R'(q=2) = 9 - 12 = -3$$

8

Panel 9

<u>Marginal Cost:</u> The marginal cost is the approx. cost of producing one additional unit.

marginal cost is $C'(q)$ or $\frac{dC}{dq}$

<u>Ex:</u> A cost function is $C(q) = 0.0001q^3 - 0.02q^2 + 5q + 5000$

Find fixed cost and marginal cost when $q = 50$

$C(0) = 5000$

$C'(q) = 0.0001 \cdot 3q^2 - 0.02 \cdot 2q + 5$

$C'(50) = $ — " — subst. $q = 50$: $\underline{\underline{+10}}$

-21

9

Panel 10

<u>Relative Rate of Change</u>

If $f(x)$ is a function, rate of change is

The rate of change relative to $f(x)$ is called the

relative rate of change $= \frac{f'(x)}{f(x)}$

<u>Ex:</u> Find relative rate of change for $y = 3x^2 - 5x + 25$ when $x = 5$.

HW

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