Panel 1

The (Calc) Story so far....

$\lim_{x \to a} f(x) = L$ as x gets closer to a, $f(x)$ gets closer to L

f continuous at $x = a$:
(1) $f(a)$ exists
(2) $\lim_{x \to a} f(x)$ exists
(3) (1) = (2)

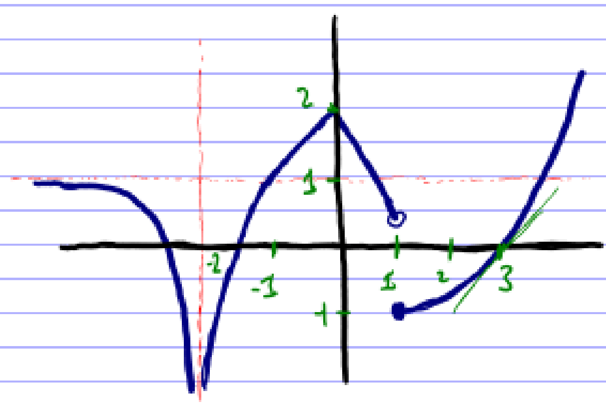
no gaps, no holes

$$f'(x) = \frac{d}{dx} f(x) = \frac{df}{dx} = \frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

slope of tangent

1

Panel 2



$f(3)$ zero

$f'(3)$ pos.

$\lim_{x \to 1^+} f(x)$ neg.

$\lim_{x \to -2^-} f(x)$ neg ∞

List points where f is not continuous at $x = -2, 1$

List points where f is not differentiable at $-2, 1, 0$

2

Panel 3

Differentiation Shortcuts

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

f' is the slope of the tangent line

Power Rule: $\frac{d}{dx}[x^n] = n x^{n-1}$

Sum Rule: $\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)$

Constant Factor: $\frac{d}{dx}(c \cdot f(x)) = c \cdot \frac{d}{dx} f(x)$

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Panel 4

Ex: $f(x) = x^4(3x^2 - 2x + 1) = 3x^6 - 2x^5 + x^4$

$f'(x) = 3 \cdot 6x^5 - 2 \cdot 5x^4 + 4x^3$

$g(x) = (2x-3)^2 = 4x^2 - 12x^1 + 9$

$g'(x) = 8x - 12 \cdot 1 \cdot x^0 = 8x - 12$

$h(x) = \left(x - \frac{1}{x}\right)^2 = x^2 - 2x\frac{1}{x} + \frac{1}{x^2} = x^2 - 2 + \frac{1}{x^2}$

$= x^2 - 2 + x^{-2}$

$h'(x) = 2x - 2x^{-3}$

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Panel 5

$y = x^3 \sqrt[3]{x} = x^3 \cdot x^{1/3} = x^{3+1/3} = \underline{x}^{10/3}$

$\underline{y' = \frac{10}{3} x^{7/3}}$

$y = \sqrt{9x} = (9x)^{1/2} = \underline{3} x^{1/2}$

$y' = 3 \frac{1}{2} x^{1/2 - 1} = \frac{3}{2} x^{-1/2}$

$(9x)^{1/2} = 9^{1/2} \cdot x^{1/2}$

$\sqrt{9 \cancel{+ 2x}}$ no good!

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Panel 6

$g(x) = \frac{1}{\sqrt[3]{8x^2}} = (8x^2)^{-1/3} = 8^{-1/3} (x^2)^{-1/3} = \boxed{\frac{1}{2} x^{-2/3}}$

$\frac{1}{\sqrt[3]{8x^2}} = \frac{1}{(8x^2)^{1/3}} = \frac{1}{8^{1/3} (x^2)^{1/3}} = \frac{1}{2 x^{2/3}} = \frac{1}{2} x^{-2/3}$

$y' = \frac{1}{2} \cdot \left(-\frac{2}{3}\right) x^{-2/3 - 1} = \underline{-\frac{1}{3} x^{-5/3}}$

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Panel 7

Name: ________

Quiz: Find the following derivatives:

a) $f(x) = x^3 \to f'(x) =$

b) $f(x) = \sqrt[3]{x} \to f'(x)$

c) $f(x) = 3x^2 - 2x + 1 \to f'(x)$

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Panel 8

d) $\frac{d}{dx}\left[5x^3 - 3\sqrt{x^5} + \pi^2\right]$

e) Find slope of tangent line to $y = 3x^2 - 7x + 1$ at $x = 1$

d) Find $f'(x)$ if $f(x) = x(x^2-1)^2$

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Panel 9

Name: ______

Quiz: Find the following derivatives:

a) $f(x) = x^4 \rightarrow f'(x) = 4x^3$

b) $f(x) = \sqrt[5]{x} \Rightarrow f'(x) = \frac{1}{5}x^{1/5 - 1} = \frac{1}{5}x^{-4/5}$

$x^{1/5}$

c) $f(x) = 4x^2 - 3x + 1 \rightarrow f'(x) = 4 \cdot 2x^1 - 3$

$8x - 3$

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Panel 10

d) $\frac{d}{dx}\left[4x^5 - 2\sqrt{x^3} + e^2\right]$ $\sqrt{x^3} = (x^3)^{1/2} = x^{3/2}$

$20x^4 - 2 \cdot \frac{3}{2}x^{3/2 - 1} = 20x^4 - 3x^{1/2}$

e) Find ~~slope~~ equation of tangent line to $y = 3x^2 - 5x + 1$ at $x = 1$

$y + (-1) = 1(x - \frac{1}{1})$ $y' = 6x - 5$ at $x = 1$: $y' = 1$

d) Find $f'(x)$ if $f(x) = x^2(x-1)^2$.

$= x^2(x^2 - 2x + 1) = x^4 - 2x^3 + x^2$

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $y' = 4x^3 - 6x^2 + 2x$

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Panel 11

More complicated differentiation Rules

Product Rule: $\frac{d}{dx}\left(f(x)\cdot g(x)\right) = f'(x)\,g(x) + f(x)\cdot g'(x)$ ~ BAD

Quotient Rule: $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{[g(x)]^2}$ ~ WORSE!

Chain Rule: $\frac{d}{dx} f(g(x)) = f'(g(x))\cdot g'(x)$ ~ confusing

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Panel 12

Exam question: use the def. of derivative to find f' if

$f(x) = 2x^2 - 7$

$f'(x) = 4x$

$$\lim_{h\to 0} \frac{\left[2(x+h)^2 - 7\right] - \left[2x^2 - 7\right]}{h} = 4x$$

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Panel 13

We will use Maple for complicated derivatives:

A) Define $f(x) := \ldots$ defines a function

B) Find derivative: $f'(x)$ (like it's)

C) Simplify by right-clicking

Ex: $f(x) = \frac{x^2(x-1)}{\sqrt{x^2+1}}$

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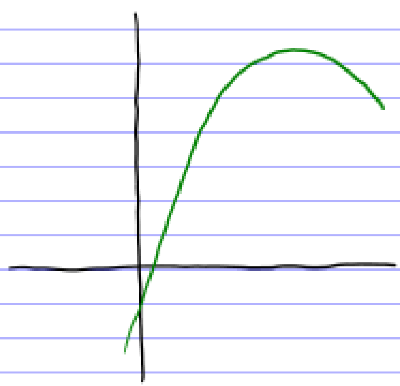
Panel 14

Why study Derivatives?

last time



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