Panel 1

Last Time

Limits at infinity: $\lim\limits_{x \to \infty} \frac{p(x)}{q(x)} = \begin{cases} \deg(p) < \deg(q) & : 0 \\ \deg(p) = \deg(q) & : \# \\ \deg(p) > \deg(q) & : \pm\infty \text{ or not a number} \end{cases}$

$\lim\limits_{x \to \infty} \frac{2x^5 - 7x}{x^4 - 9x^2}$

The derivative $f'(x) = \lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$

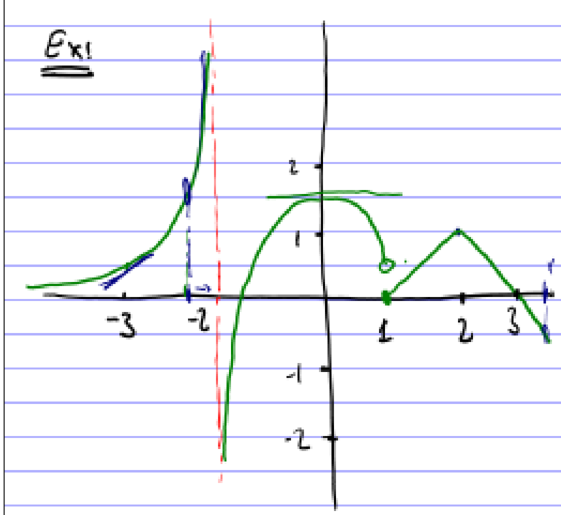
geometric interpretation: slope of tangent line.

Notation $f'(x) = \frac{d}{dx} f(x) = \frac{df}{dx}$

1

Panel 2

Ex:



a) $\lim\limits_{x \to 3} f(x) = 0$

b) $\lim\limits_{x \to 1^+} f(x) = 0$

c) $\lim\limits_{x \to -2^-} f(x) = +\infty$, d.n.e

d) $\lim\limits_{x \to -\infty} f(x) = 0$

e) $\lim\limits_{x \to 0} f(x) \approx 1.5$

g) $\lim\limits_{x \to 1} f(x) =$ d.n.e.

h) $f'(-3) > 0$

i) $f'(3) < 0$

i) $f'(0) = 0$

k) $f'(2) =$ d.n.c.

2

Panel 3

Ex:
$$f(x) = \begin{cases} \dfrac{x^2-9}{x^2-x-6} & \text{if } x \neq 3 \\ K & \text{if } x = 3 \end{cases}$$

What should k be, if anything, so that f(x) is continuous at x = 3 ?

(1) $f(3) = K$

(2) $\lim_{x\to 3} f(x) = \lim_{x\to 3} \dfrac{x^2-9}{x^2-x-6} = \lim_{x\to 3} \dfrac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}(x+2)} = \underline{\underline{6/5}}$

(3) (1) = (2) $\boxed{k = 6/5}$

3

Panel 4

499)

$\lim_{x\to\infty} \dfrac{6-4x^2+x^3}{4+5x-7x^2} \sim \lim_{x\to\infty} \dfrac{x^3}{-7x^2} = \lim_{x\to\infty} -\frac{1}{7}x^1 = -\infty$

$y = x^2+4$ find slope of tangent at $(-2,8)$

4

Panel 5

Name: __________

Quiz:

① Find the following limits:

a) $\lim\limits_{x \to 1} \dfrac{x^2-1}{x}$

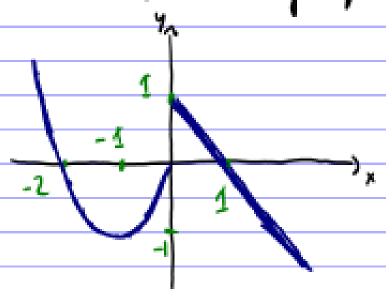
b) $\lim\limits_{x \to 3} \dfrac{x^2-x-6}{x^2-9}$

c) $\lim\limits_{x \to \infty} \dfrac{x^2-7x-9}{5-4x-3x^2}$

5

Panel 6

② Find the following quantities:

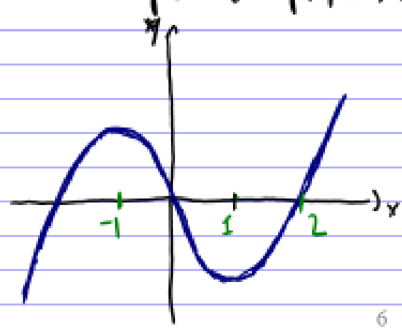


a) $\lim\limits_{x \to -2} f(x)$

b) $\lim\limits_{x \to 0^-} f(x)$

c) $\lim\limits_{x \to 0} f(x)$

③ Find the signs (neg, pos, zero) of the quantities:



a) $f'(-1)$

b) $f'(0)$

c) $f'(2)$

6

Panel 7

Quiz

Name: ____________

① Find the following limits:

a) $\lim\limits_{x \to 1} \dfrac{x^2-2}{x}$

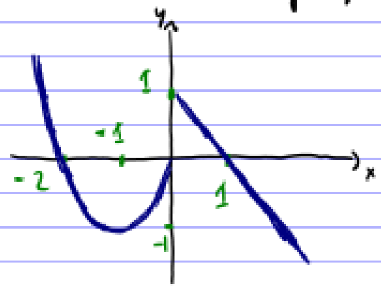
b) $\lim\limits_{x \to 3} \dfrac{x^2-9}{x^2-x-6}$

c) $\lim\limits_{x \to \infty} \dfrac{x^2-7x-9}{5-4x-8x^2}$

7

Panel 8

② Find the following quantities:

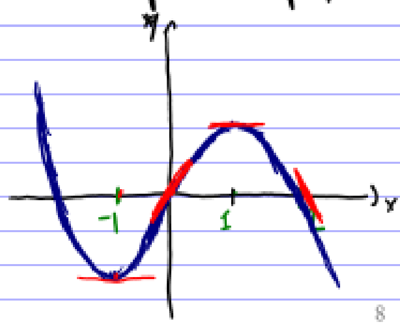


a) $\lim\limits_{x \to 1} f(x)$

b) $\lim\limits_{x \to 0^+} f(x)$

c) $\lim\limits_{x \to 0} f(x)$

③ Find the signs (neg, pos, zero) of the quantities:



a) $f'(0)$

b) $f'(1)$

c) $f'(2)$

8

Panel 9

Back to Derivatives:

$f'(0) = 6 \cdot 0 = 0$

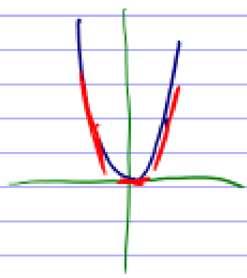
$f'(1) = 6 \quad , f'(-1) = -6$

Ex: $f(x) = 3x^2$. Find $f'(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} =$$

$$= \lim_{h \to 0} \frac{3(x+h)^2 - 3x^2}{h} = \lim_{h \to 0} \frac{3(x^2 + 2xh + h^2) - 3x^2}{h} =$$

$$= \lim_{h \to 0} \frac{6xh + 3h^2}{h} = \lim_{h \to 0} \frac{\cancel{h}(6x + 3h)}{\cancel{h}}$$



$$= \lim_{h \to 0} 6x + 3h = 6x$$

9

Panel 10

Find $\frac{d}{dx} f$, where $f(x) = \frac{1}{x}$ $\quad , f'(x) = -\frac{1}{x^2}$

$$\frac{d}{dx} f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} =$$

$$= \lim_{h \to 0} \frac{\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)}}{h} =$$

$$= \lim_{h \to 0} \frac{x - (x+h)}{x(x+h)} \cdot \frac{1}{h} = \lim_{h \to 0} \frac{-\cancel{h}}{x(x+h) \cdot \cancel{h}}$$

$$= \lim_{h \to 0} \frac{-1}{x(x+h)} = -\frac{1}{x^2}$$

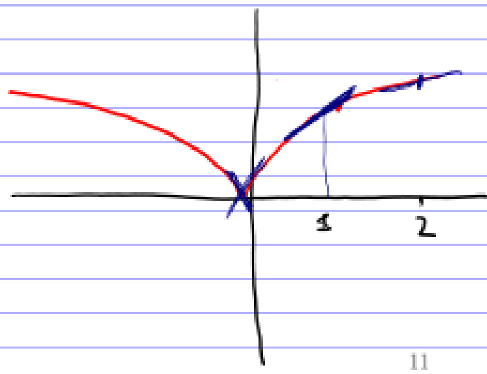
10

Panel 11

Derivatives are defined via limits. Thus, they do not necessarily have to exist:

<u>Def</u>: If the graph of a function does not have a unique tangent line at a point, it is not differentiable at that point.

<u>Ex</u>:



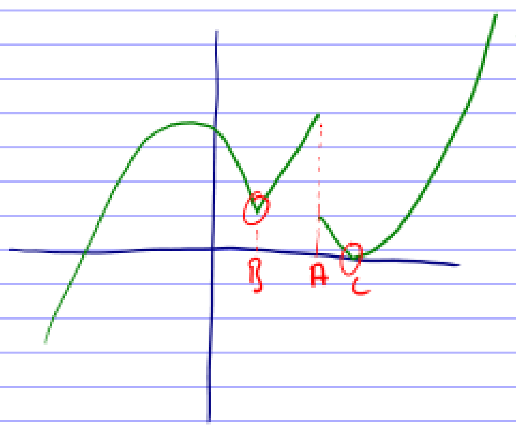
a) $f'(2) > 0$

b) $f'(1) >> 0$

c) $f'(0)$ <u>d.n.e.</u>

11

Panel 12



Not continuous at $x = A$

Not diff'ble at $x = B$ and C

12

Panel 13

Derivatives as limits can get complicated $\Rightarrow$ need short cut:

Ex: $f(x) = x^1 \Rightarrow f'(x) = 1 \cdot x^0$

$\lim_{h \to 0} \frac{x+h-x}{h} =$

$f(x) = x^2 \Rightarrow f'(x) = 2x^1$

$\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h} = \lim \frac{2xh + h^2}{h} = \frac{h(2x+h)}{h} = \underline{\underline{2x}}$

$f(x) = x^3 \Rightarrow f'(x) = 3x^2$ verify at home

$f(x) = x^4 \Rightarrow f'(x) = 4x^3$

$f(x) = x^n \Rightarrow f'(x) = n x^{n-1}$ Power Rule

13

Panel 14

Rules for Differentiation - Part I

The Power Rule: $f(x) = x^n \Rightarrow f'(x) = n x^{n-1}$

The Constant Rule: $f(x) = c \Rightarrow f'(x) = 0$

The Constant Factor Rule: $\frac{d}{dx}\left[c \cdot f(x)\right] = c \cdot f'(x)$

14

Panel 15

Examples: $f(x) = 3 \Rightarrow f'(x) = 0$

$g(x) = x^2 \Rightarrow f'(x) = 2x$

$h(x) = 3x^5 \Rightarrow f'(x) = 3 \cdot 5x^4 = 15x^4 = \lim_{h\to 0} \frac{3(x+h)^5 - 3x^5}{h}$

$x^{1/2} = k(x) = \sqrt{x} \Rightarrow f'(x) = \frac{1}{2} x^{-1/2} = \frac{1}{2} \frac{1}{x^{1/2}} = \frac{1}{2} \frac{1}{\sqrt{x}}$

$\frac{1}{2} x^{1/2 - 1}$ $\quad \frac{9}{5} x^{1/3} = l(x) = \frac{9}{5} \sqrt[3]{x} \Rightarrow f'(x) = \frac{9}{5} \cdot \frac{1}{3} x^{-2/3}$

$4x^{-2/3} = m(x) = \frac{4}{\sqrt[3]{x^2}} \Rightarrow f'(x) = 4(-\frac{2}{3}) x^{-5/3}$

$n(x) = 3\pi e^2 \Rightarrow f'(x) = 0$

15

Panel 16

More (simple) Differentiation Rules

The Sum/Difference Rule: $\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$

Ex: $f(x) = x^2 + 3x^1 - 7$ $\quad = \lim_{h\to 0} \frac{(x+h)^2 + 3(x+h) - 7 - (x^2 + 3x - 7)}{h}$

$f'(x) = 2x + 3 \cdot 1 x^0 - 0 = 2x + 3$

$g(x) = 5x^2 - \frac{7}{x^2} + 9\sqrt[3]{x^4} + \pi^2$

$= 5x^2 - 7x^{-2} + 9x^{4/3} + \pi^2$

$g'(x) = 10x + 14x^{-3} + 9 \cdot \frac{4}{3} x^{1/3} + 0$

16

Panel 17

More Examples:

A) $f(x) = x^5$

B) $f(x) = x^3 + 9$ $3x^2$

C) $f(x) = 3x^2 - 7x$

D) $f(x) = 1/x^3 = x^{-3}$ $f'(x) = -3x^{-4}$

E) $f(x) = 3x^2 - \frac{7}{x} + \sqrt{x} = 3x^2 - 7x^{-1} + x^{\frac{1}{2}}$, $f'(x) = 6x + 7x^{-2} + \frac{1}{2}x^{-1/2}$

F) $f(x) = 9x^3 - 8\sqrt{x^3} + 5\sqrt[3]{x^2} + \pi$

$= 27x^2 - 8 \cdot \frac{3}{2} x^{1/2} + 5 \frac{2}{3} x^{-1/3} + 0$

Quiz on Wed

17