Math 1303
Final Exam Review

The following problems are reflective of the types of problems on the final examination. You should work out these problems to supplement studying for the final exam. Please be aware that there may be some types of problems on the final exam which are not on this sheet, and some types of problems on this sheet which are not on the final exam.

1. Find the following limits.

a) $\lim_{x\to\infty}\frac{3x-x^3}{2x^3+1}$

b) $\lim_{x\to-2}\frac{x-3}{x^2-x-6}$

c) $\lim_{x\to-5}(-x^2+x)$

2. a) $f(x)=\begin{cases} 6 & \text{for } x=1 \\ \dfrac{x^2+4x-5}{x-1} & \text{for } x\neq 1 \end{cases}$

b) $g(x)=\begin{cases} 1 & \text{for } x<0 \\ x^2 & \text{for } x\geq 0 \end{cases}$

(Use the definition of continuity for #2. Also, graph part (b).)

3. Use the definition to find the derivative of the following.

a) $f(x)=-7x^2+4x$

b) $f(x) = 5 - 2x$

4. Differentiate.

a) $f(x) = e^x + x^3 - 5\ln x$

b) $g(q) = 8q^5 + 7q^4 - \dfrac{4}{7q^3}$

c) $y = \sqrt[5]{x} - \dfrac{2}{x} + 4e^x$

d. Find $\dfrac{d^3y}{dx^3}$ for $y = 2x^5 + 3x^3 + 9x - 5e^x$

5. Sketch the graphs of the following. Find critical points and points of inflection. State where the graph is increasing, decreasing, concave up and concave down.

a) $f(x) = x^3 - 3x + 2$

b) $y = -\frac{4}{3}x^3 + 2x^2 - x$

6. Integrate.

a) $\int (e^x - x^2 - 5)dx$

d) $\int_{-1}^{1} (t^2 - t^4)dt$

b) $\int_{1}^{e} (x^2 - \frac{1}{x})dx$

e) find $f(x)$ if $f'(x) = \sqrt{x} - 3$ and $f(4) = -1$. .

c) $\int (\frac{6}{x} + \frac{4}{x^4} - \sqrt{x})dx$

7. Solve using Gauss-Jordan elimination:

$$\begin{aligned} x + 2y - z &= -4 \\ \text{a) } x - y + 2z &= 14 \\ 2x + 2y - z &= -1 \end{aligned}$$

Goal : eliminate ○ then ○ then ○ variable

$$\begin{aligned} x + 2y - z &= -4 \\ 3y - 3z &= -18 \\ 2x + 2y - z &= -1 \end{aligned} \Rightarrow \begin{aligned} x + 2y - z &= -4 \\ 3y - 3z &= -18 \\ -2y + z &= 7 \end{aligned} \quad \begin{aligned} &1.2 \Rightarrow \\ &1.3 \end{aligned}$$

$$\Rightarrow \begin{aligned} x + 2y - z &= -4 \\ 6y - 6z &= -36 \\ -6y + 3z &= 21 \end{aligned} \Rightarrow \begin{aligned} x + 2y - z &= -4 \\ 6y - 6z &= -36 \\ -3z &= -15 \end{aligned}$$

$x - 2 - 5 = -4 \Rightarrow \underline{\underline{x = 3}}$

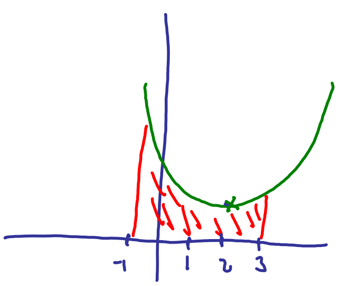
$6y - 30 = -36 \Rightarrow \underline{\underline{y = -1}}$

$\Rightarrow \underline{\underline{z = 5}}$

$\underline{\underline{x = 3}},\ \underline{\underline{y = -1}},\ \underline{\underline{z = 5}}$

$$\begin{aligned} &3x+y+z=2 \\ \text{b) } &x-y+z=0 \\ &-x+4y-2z=3 \end{aligned}$$

8. Find the area under the curve $y=x^2-4x+5$ from x=-1 to x=3. Sketch the region.



vertex $-b/2a = x = \frac{4}{2} = 2,\ y = 1$

$\int_{-1}^{3} x^2-4x+5\,dx = \frac{1}{3}x^3 - 2x^2 + 5x \Big|_{-1}^{3} = \frac{1}{3}\cdot 27 - 18 + 15 - \left(-\frac{1}{3} - 2 - 5\right) =$

$= 9 - 18 + 15 + \frac{1}{3} + 7 = 13 + \frac{4}{3} = \underline{\underline{\frac{40}{3}}}$

9. If $R(x)=60x-.3x^2$ and $C(x)=12x+2$, find the maximum profit and the number of units that must be produced and sold to maximize the profit.

$P(x) = R(x) - C(x) = 60x - 0.3x^2 - (12x+2)$ ← downwards parabola so critical point is max.

$= \underline{48x - 0.3x^2 - 2}$

$P'(x) = 48 - 0.6x = 0 \Rightarrow \underline{\underline{x = 80}}$

Produce $x=80$ units for a max profit of $\underline{\underline{P(80)}} = \underline{\underline{1918}}$

10. Suppose \$8000 is invested in an account. How much money is in the account in 6 years if the interest rate is 5% compounded: a) monthly b) continuously?

a) $S = 8000\left(1 + \frac{0.05}{12}\right)^{6\cdot 12} = \underline{\underline{10792.14}}$

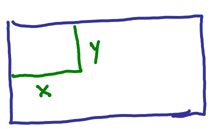
b) $S = 8000\, e^{0.05\cdot 6} = 8000\, e^{0.3} = \underline{\underline{10798.87}}$

11. An object is dropped from a certain height. It is known that it will fall a distance of $s(t) = 16t^2$ where s is in feet and t is in seconds. What is the average speed from 3 to 5 seconds.

speed = distance/time

$\Rightarrow$ avg speed $= \dfrac{s(5) - s(3)}{5-3} = \dfrac{16\cdot 25 - 16\cdot 9}{2} = \dfrac{16\cdot 16}{2} = 8\cdot 16 = \underline{\underline{128}}$

12. A rancher has 50 feet of fencing to fence off a rectangular animal pen in the corner of a barn. The corner of the barn will not be fenced. What dimensions of the rectangle will maximize the area?



$x + y = 50 \Rightarrow y = 50 - x$

downwards parabola

$A = xy = x(50-x) = 50x - x^2$

$A'(x) = 50 - 2x = 0 \Rightarrow \underline{\underline{x = 25}}$ yields max

Dimensions $\underline{x = 25}$, $\underline{y = 25}$ for max area of $\underline{625}$ ft^2

13. After t hours of operation, a coal mine is producing coal at a rate of $40 + 2t - 9t^2$ tons of coal per hour. Find the formula for the output of the coal mine after t hours of operation if we know that after 2 hours, 80 tons have been mined.

Rate of production $= 40 + 2t - 9t^2$

$\Rightarrow$ Production $= \int 40 + 2t - 9t^2\,dt = \underline{40t + t^2 - 3t^3 + C = P(t)}$

know $P(2) = 80 = 80 + 4 - 24 + C \Rightarrow C = \underline{\underline{20}}$

$\underline{\underline{P(t) = 40t + t^2 - 3t^3 + 20}}$

14. A firm estimates that it will sell N units of a product after spending x dollars on advertising, where $N(x) = -x^2 + 300x + 6$ and x is measured in thousands of dollars. What is the rate of change of the number of units sold with respect to the amount spent on advertising after spending \$10 thousand?

Rate of change is derivative $N'(t) = -2x + 300$

$\rightarrow N'(10) = -20 + 300 = \underline{\underline{280}}$

15. Use Maple to solve the following:

a) Differentiate

1. $f(x) = \dfrac{3x^2 - 5x}{x^2 - 1}$

$f(x) := \dfrac{3x^2 - 5x}{x^2 - 1}$

$x \rightarrow \dfrac{3x^2 - 5x}{x^2 - 1}$

$simplify(f'(x))$

$\dfrac{-6x + 5x^2 + 5}{(x^2 - 1)^2}$

2. $y = (x-4)^4(2x+3)^7$

$f(x) := (x-4)^4 \cdot (2x+5)^7$

$x \rightarrow (x-4)^4 (2x+5)^7$

$f'(x)$

$4(x-4)^3(2x+5)^7 + 14(x-4)^4(2x+5)^6$

3. $f(q) = \ln(e^q - 2)$

4. $y = \dfrac{x^2(3x-4)^7}{e^{3x}}$, find $\dfrac{d^3y}{dx^3}$ in its factored form.

$f(x) := \dfrac{x^2 \cdot (3x-4)^7}{e^{3x}}$

$x \rightarrow \dfrac{x^2(3x-4)^7}{e^{3x}}$

$factor(f'''(x))$

$-\dfrac{9(3x-4)^4(-352 - 1053x^4 + 3360x - 7320x^2 + 4536x^3 + 81x^5)}{e^{3x}}$

b) Find the slope of the line tangent to the curve $y = x\sqrt{x^2+5}$ at $x = 2$.

$f(x) := x \cdot \sqrt{x^2 + 5}$

$x \rightarrow x\sqrt{x^2+5}$

$f'(2)$

$\dfrac{13}{9}\sqrt{9}$

c) Let $g(x) = x^3 + x^2 - 5x + 3$.

1. Find all critical points and points of inflection.

2. Graph g(x) and from the graph and the answer to part (1), state the intervals over which g(x) is increasing and decreasing, concave up and concave down.

d) Integrate

1. $\int x^2 e^x dx$

$$int\left(x^2 \cdot e^x, x\right)$$

$$\left(2 - 2x + x^2\right) e^x$$

2. $\int_0^1 \frac{y+1}{(y^2+2y+6)^2} dy$

$$int\left(\frac{y+1}{\left(y^2+2y+6\right)^2}, y = 0..1\right)$$

$$\frac{1}{36}$$

16. Use Excel to solve the following problems from the textbook:
p. 213 #24; p. 216 #13; p. 227 # 7, 11, 19; p. 242 # 5, 17

Review sheet answers:

1. a) -1/2 b) does not exist c) -30

2. a) continuous at x=1 b) discontinuous at x=0.

3. a) -14x+4 b) -2

4. a) $f'(x) = e^x + 3x^2 - \dfrac{5}{x}$

   b) $g'(q) = 40q^4 + 28q^3 + \dfrac{12}{7q^4}$

   c) $y' = \dfrac{1}{5\sqrt[5]{x^4}} + \dfrac{2}{x^2} + 4e^x$

   d) $\dfrac{d^3y}{dx^3} = 120x^2 + 18 - 5e^x$

5. a) b)

6. a) $e^x - \dfrac{x^3}{3} - 5x + C$

   b) $\dfrac{e^3 - 4}{3} \approx 5.36$

   c) $6\ln x - \dfrac{4}{3x^3} - \dfrac{2}{3}\sqrt{x^3} + C$

6. d) $\dfrac{4}{15}$

   e) $f(x) = \dfrac{2}{3}\sqrt{x^3} - 3x + \dfrac{17}{3}$

7. a) (3,-1,5) b) (-1,2,3)

8. $\frac{40}{3}$ units ^{2}

9. 80 units , \$1918

10. a) \$10,792.14 b) \$10,798.87

11. 128 ft/sec

12. 25 ft x 25 ft

13. $P = -3t^3 + t^2 + 40t + 20$

14. 280 units

15. a)

1. $f'(x) = \frac{(6x-5)}{(x^2-1)} - 2\frac{(3x^2-5x)x}{(x^2-1)^2} = \frac{5x^2-6x+5}{(x^2-1)^2}$
2. $f'(x) = 4(x-4)^3(2x+3)^7 + 14(x-4)^4(2x+3)^6$
3. $f'(x) = \frac{e^x}{e^x - 2}$
4. $\frac{d^3y}{dx^3} = -9\frac{(3x-4)^4(-352+3360x+4536x^3-7320x^2-1053x^4+81x^5)}{e^{3x}}$

b) $\frac{13}{3}$

c) 1. c.p.: $\left(-\frac{5}{3}, \frac{256}{27}\right)$, (1,0) p.o.i.: $\left(-\frac{1}{3}, \frac{128}{27}\right)$

2. increasing: $\left(-\infty, -\frac{5}{3}\right)$ U $(1, \infty)$ decreasing: $\left(-\frac{5}{3}, 1\right)$

concave up: $\left(-\frac{1}{3}, \infty\right)$ concave down: $\left(-\infty, -\frac{1}{3}\right)$

d) 1. $x^2e^x - 2xe^x + 2e^x + C$

2. $\frac{1}{36} = .0278$

16. p. 213, #24, 6.80%
p. 216, #13, \$14,091.10
p. 227, #7, \$29,984.06; #11, \$90,231.01; #19, \$1332.73
p. 242 #5, (a) \$221.43, (b) \$25, (c) \$196.43; #17, nper=23.956, therefore, 23 full payments