Math 1303 - Practice Exam 3

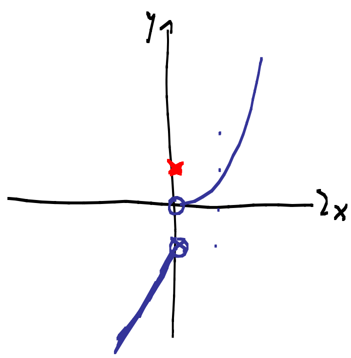
PRELIMINARY !

1. Sketch the graph for $f(x) = \begin{cases} x^2 & for \quad x > 0 \\ 1 & for \quad x = 0 \\ 3x-1 & for \quad x < 0 \end{cases}$

There are 3 things to graph: x^2 (parabola) for $x>0$

1 for $x=0$ (a point)

$3x-1$ (line, slope 3, y-int. -1) for $x<0$



(not on Exam 3 but on final)

2. The Consumer Price Index (CPI) of an economy is described by the function $I(t) = 200 + 3t - 0.4t^2$, where t is time in years and t = 0 corresponds to the year 2000.
   a) Find the average change in the CPI from 2001 to 2003.
   b) Find the instantaneous rate of change in the CPI with respect to time in 2005. Interpret your result.

a) Average change is $\frac{I(3)-I(1)}{3-1} = \frac{2.8}{2} = 1.4$

$I(3)$ is CPI for year 2003

$I(1)$ is CPI for year 2001

b) Inst. rate of change is derivative: $I'(t) = 3 - 0.8t$

$\Rightarrow I(5) = 3 - 0.8 \cdot 5 = -1$

(not on Exam 3 but on final)

3. The analysis of the daily output of a factory assembly line shows that about $H(t) = 60t + t^2 - t^3$ units are produced after t hours of work, $0 \le t \le 8$.
   a) Find the average change in production as t changes from 3 to 5 hours.
   b) Find the instantaneous rate of change of production when t = 4 hours.

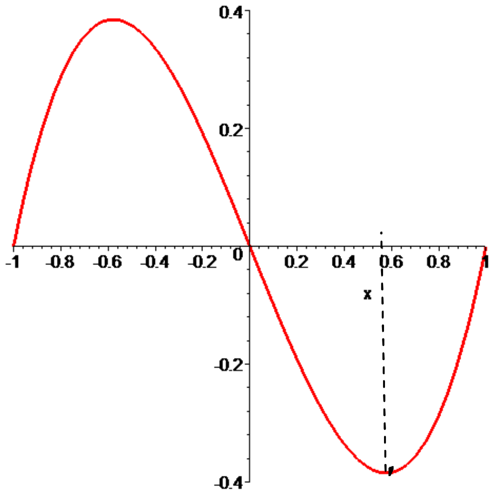
a) average change is $\frac{H(5)-H(3)}{5-3} = 19$

b) inst. rate of change is derivative: $H'(t) = 60 + 2t - 3t^2$

$\Rightarrow H(4) = 60 + 2 \cdot 4 - 3 \cdot 4^2 = 20$

not on exam 3 but on final

4. Consider the graph shown below and answer the following questions:



f'(0.55) zero

f''(0.55) positive

f'(0) negative

f''(0) zero

f is **decreasing** *on the interval(s):*

(-0.55, 0.55)

f is **concave down** *on the interval(s):*

(-1, 0)

Also possible questions

$\int_{-1}^{0} f(x)\,dx$ = pos

$\int_{0}^{1} f(x)\,dx$: neg.

$\int_{-1}^{1} f(x)\,dx$ = zero

5. Differentiate and simplify:

a) $y = 6x^5 - 9\ln(x) - \frac{2}{x^3}$ $= 6x^5 - 9\ln(x) - 2x^{-3}$

$\underline{y' = 30x^4 - 9/x + 6x^{-4}}$

b) $f(q) = 8e^q + 6\sqrt[3]{q} + 21$ $= 8e^q + 6q^{1/3} + 21$

$\underline{f'(q)} = 8e^q + 6 \cdot \frac{1}{3} q^{-2/3} = \underline{8e^q + 2q^{-2/3}}$

c) $y = 6e^x - 3x^5 - 13$

$y' = 6e^x - 15x^4$

d) $f(t) = 4\ln t + \sqrt[5]{t^2}$ $= 4\ln(t) + t^{2/5}$

$f'(t) = 4/t + \frac{2}{5} t^{-3/5}$

6. Investigate concavity, i.e. intervals where f is concave up or down, of

a) $f(x) = x^3 + x^2 - 5x - 5$

$f'(x) = 3x^2 + 2x - 5$

$f''(x) = 6x + 2 = 0$

$x = -1/3$

		$-1/3$	
f''	$-$		$+$
f	$\cap$		$\cup$

concave down: $(-\infty, -1/3)$

concave up $(-\frac{1}{3}, \infty)$

b) $f(x)=12+2x^2-x^4$

$f'(x) = 4x - 4x^3$

$f''(x) = 4 - 12x^2 = 0$

$\Rightarrow x = \pm\sqrt{1/3}$

		$-\sqrt{1/3}$	$+\sqrt{1/3}$
f''	$-$	$+$	$-$
f	$\cap$	$\cup$	$\cap$
	down	up	down

7. Find

$$\int(3x^8+6e^x+\frac{7}{x})dx = 3\,\frac{1}{9}x^9 + 6e^x + 7\ln|x| + C$$

$$\int(5e^x+20x^4-\frac{2}{x})dx. = 5e^x + 20\,\frac{1}{5}x^5 - 2\ln|x| + C$$

8. Evaluate

$$\int_1^2(8x^3-3)dx = 8\,\frac{1}{4}x^4 - 3x\Big|_1^2 = 2x^4-3x\Big|_1^2 =$$

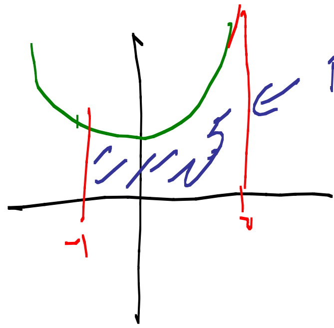
$$= (2\cdot 4-6)-(2-3) =$$

$$= 12-6+1 = 7$$

$$\int_1^3(6x^2-1)dx = 2x^3-x\Big|_1^3 = (2\cdot 27-3)-(2-1) =$$

$$= 54-3-1 = 50$$

9. Find the area under the graph $y = 2x^2 + 1$ from x = -1 to x = 2. Sketch and shade the region.

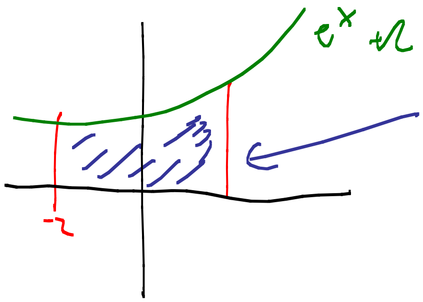


$$\text{Area} = \int_{-1}^{2} 2x^2 + 1\,dx = \frac{2}{3}x^3 + x \Big|_{-1}^{2} =$$

$$= \left(\frac{2}{3}8 + 2\right) - \left(-\frac{2}{3} - 1\right) =$$

$$= \frac{16}{3} + 2 + \frac{2}{3} + 1 = \frac{18}{3} + 3 = \underline{\underline{9}}$$

Do the same for the area under the graph $y = e^x + 2$ from x = -2 to x = 1.



$$\text{Area} = \int_{-2}^{1} e^x + 2\,dx = e^x + 2x \Big|_{-2}^{1} =$$

$$= \left(e^1 + 2\right) - \left(e^{-2} - 4\right) =$$

$$= \underline{e - 1/e^2 + 6}$$

10. Suppose the marginal cost for producing the q^{th} item is $C' = 5e^q - 18q^2 - 320$, and the fixed cost is $1700. Find the formula for the cost function.

$$C'(x) = 5e^q - 18q^2 - 320$$

$$\Rightarrow C(x) = \int C'(x)\,dx = 5e^q - 6q^3 - 320q + c$$

Also: $C(0) = 1700$ (fixed cost)

$$\Rightarrow 5e^0 - 6\cdot 0^3 - 320\cdot 0 + c = 1700$$

$$5 + c = 1700 \Rightarrow c = \underline{1695}$$

$$\Rightarrow C(q) = \underline{5e^q - 6q^3 - 320q + 1695}$$

11. Suppose the marginal cost of making q throw rugs is $c' = 8q - 3\sqrt{q} + 4e^q$, and the fixed cost is \$4400. Find the formula for the cost function.

$$C'(q) = 8q - 3q^{1/2} + 4e^q$$

$$\Rightarrow C(q) = \int C'(q)\,dq = 4q^2 - 3\tfrac{2}{3}q^{3/2} + 4e^q + C$$

$$= 4q^2 - 2q^{3/2} + 4e^q + C$$

$$\Rightarrow C(0) = 0 - 0 + 4 + c = 4400 \Rightarrow c = \underline{4396}$$

$$\Rightarrow \underline{C(q) = 4q^2 - 2q^{3/2} + 4e^q + 4396}$$

12. A supermarket manager wants to establish an inventory policy for frozen orange juice. He finds that his inventory costs each month are $C(x) = \dfrac{360000}{x} + 4x$ dollars, where x is the number of cases of orange juice. How many cases should he order each month to minimize his inventory costs?

$$C(x) = \frac{360\,000}{x} + 4x = 360\,000x^{-1} + 4x$$

$$C'(x) = -360\,000x^{-2} + 4 = 0$$

$$\Rightarrow 4 = 360\,000\frac{1}{x^2}$$

$$\Rightarrow 4x^2 = 360\,000 \Rightarrow x^2 = 90\,000$$

$$\Rightarrow x = \pm 300 \quad \text{(0 is also special)}$$

	-300	0	300	
C'	$+$	$-$		$+$
C	$\nearrow$	$\searrow$		$\nearrow$

$\Rightarrow x = 300$ is a minimum

$\Rightarrow$ Order 300 cases for minimal cost

13. Carefully sketch the graph of $y = 2x^3 + 3x^2 - 12x - 3$. Identify all critical points and points of inflection. State the intervals over which the graph is increasing, decreasing, concave up and concave down. Identify any absolute or relative extrema.

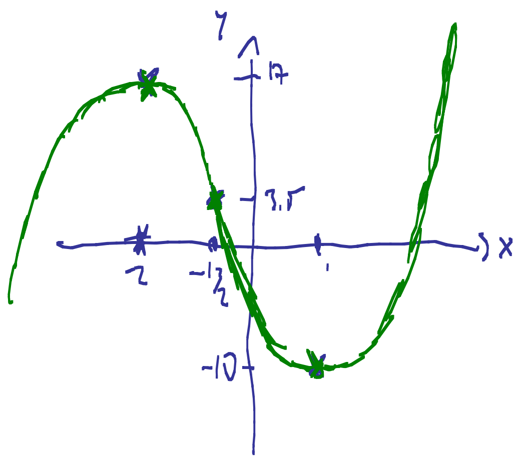
$$y' = 6x^2 + 6x - 12 =$$
$$= 6(x^2 + x - 2) =$$
$$= 6(x+2)(x-1) = 0 \Rightarrow x = -2, 1$$

$$y'' = 12x + 6 =$$
$$= 6(2x+1) = 0 \Rightarrow x = -1/2$$

$-2 \quad -1/2 \quad 1$

	-3	-1	0	2
f'	+	−	−	+
f''	−	−	+	+
f				

$$y(-2) = 17$$
$$y(-\tfrac{1}{2}) = 3.5$$
$$y(1) = -10$$



Do the same for the graph of $y = x^3 - 9x^2 + 15x - 4$.

$$y' = 3x^2 - 18x + 15 =$$
$$= 3(x^2 - 6x + 5) =$$
$$= 3(x-5)(x-1) = 0, \ x = 5, 1$$

$$y'' = 6x - 18 = 0 \Rightarrow x = 3$$

$$y(1) = 3$$
$$y(3) = -13$$
$$y(5) = -29$$

$1 \quad 3 \quad 5$

	0	2	4	6
f'	+	−	−	+
f''	−	−	+	+
f