Math 1303 Practice Exam 2

Evaluate the following limits:

$$\lim_{x\to 0}\frac{x-3}{x^2+1} = \frac{0-3}{0+1} = -3/1 = -3$$

$$\lim_{x\to 2}\frac{2-x}{x^2} = \frac{2-2}{2^2} = \frac{0}{4} = 0$$

$$\lim_{x\to 1}\frac{x-5x+6}{x-2} = \frac{1-5+6}{1-2} = \frac{2}{-1} = -2$$

$$\lim_{x\to 2}\frac{x-5x+6}{x-2} \left(= \frac{0}{0}\right) = \lim_{x\to 2} \frac{\cancel{(x-2)}(x-3)}{\cancel{x-2}} = 2-3 = -1$$

$$\lim_{x\to 3}\frac{x^2-9}{x-3} \left(= \frac{0}{0}\right) = \lim_{x\to 3} \frac{(x+3)\cancel{(x-3)}}{\cancel{x-3}} = 6$$

$$\lim_{x\to 4} \frac{x^2-16}{x^2-x-12} = \left(\frac{0}{0}\right) = \lim_{x\to 4} \frac{\cancel{(x-4)}(x+4)}{(x+3)\cancel{(x-4)}} = \frac{8}{7}$$

$$\lim_{x\to\infty}\frac{2-x}{x^2+9} = 0 \quad \text{(bottom power wins)}$$

$$\lim_{x\to\infty}\frac{3x^2-2x+7}{3-2x^2} = -\frac{3}{2} \quad \text{(even race)}$$

$$\lim_{x\to -\infty}\frac{2+3x^3}{x^2+x+1} \approx \frac{3x^3}{x^2} = \frac{3x}{1} \to \infty \quad \text{(top-power wins)}$$

Let $f(x) = \begin{cases} x^2 & for \quad x>0 \\ 1 & for \quad x=0 \\ 3x-1 & for \quad x<0 \end{cases}$ Find the limits

$\lim_{x\to -\infty} f(x)$, $= \lim_{x\to -\infty} 3x-1 = -\infty$

$\lim_{x\to 0^-} f(x)$, $= \lim_{x\to 0^-} (3x-1) = -1$

$\lim_{x\to 0^+} f(x)$, $= \lim_{x\to 0^+} x^2 = 0$

$\lim_{x \to 0} f(x)$ = d.n.e. because left + right handed limits are different

Let $f(x) = \begin{cases} 2x - 5 & , x < 2 \\ -x & , x \geq 2 \end{cases}$

Is f(x) is continuous at x = 0? At 0 there is no problem. $f(x) = 2x-5$ for x close to zero. So cont. at 0

How about at x = 2?

$f(2) = -2$, $\lim_{x \to 2^+} f(x) = -2$

$\lim_{x \to 2^-} f(x) = 2 \cdot 2 - 5 = -1$

$\Bigg\} \lim_{x \to 2} f(x)$ d.n.e. $\Rightarrow$ Not cont. at $x=2$

Let $f(x) = \begin{cases} \frac{x^2-9}{x-3} & if \quad x \neq 3 \\ 6 & if \ x = 3 \end{cases}$

Is f(x) is continuous at x = 0? At zero f is certainly continuous

How about at x = 3?

$f(3) = 6$, $\lim_{x \to 3} \frac{x^2-9}{x-3} = \lim_{x \to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}} = 6$ and therefore $\lim_{x \to 3} f(x) = f(3)$

so <u>yes</u>, cont. at $x=3$

Using the <u>definition of the derivative</u>, find
$f'(x)$ if $f(x) = -x^2 + 5x + 2$.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{-(x+h)^2 + 5(x+h) + 2 - (-x^2 + 5x + 2)}{h} = \lim_{h \to 0} \frac{\cancel{-x^2} - 2xh - h^2 + \cancel{5x} + 5h + \cancel{2} + \cancel{x^2} - \cancel{5x} - \cancel{2}}{h} =$$

$$= \lim_{h \to 0} \frac{\cancel{h}(-2x - h + 5)}{\cancel{h}} = \lim_{h \to 0} -2x - h + 5 = \underline{\underline{-2x+5}}$$

Then find the equation of the tangent line to f(x) at x = 3

Know: $f'(x) = -2x + 5$

$\Rightarrow f'(3) = -6 + 5 = -1$

$\Rightarrow$ tangent line: $\underline{y - 8 = -1(x-3)}$

$f(3) = -9 + 15 + 2 = 8$

$f'(x)$ if $f(x) = x^2 - 6x + 3$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - 6(x+h) + 3 - (x^2 - 6x + 3)}{h} = \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{6x} - 6h + \cancel{3} - \cancel{x^2} + \cancel{6x} - \cancel{3}}{h}$$

$$= \lim_{h \to 0} \frac{h(2x + h - 6)}{h} = \lim_{h \to 0} 2x + h - 5 = \underline{\underline{2x-5}}$$

$f'(x) = 2x-5 \Rightarrow f'(2) = -1$

Then find the equation of the tangent to f(x) at x = 2 $\quad f(2) = 4 - 12 + 3 = -5$

$\Rightarrow \underline{y + 5 = -1(x-2)}$

Differentiate and simplify

$y = -3x^5 - 13$

$y' = \underline{15x^4}$

$f(t) = \frac{4}{t} + \frac{t}{4} + \sqrt[5]{t^2} = 4t^{-1} + \frac{1}{4}t + t^{2/5}$

$\Rightarrow \underline{f'(t) = -4t^{-2} + \frac{1}{4} + \frac{2}{5}t^{-3/5}}.$

Consider the function $y = 2x^3 + 3x^2 - 12x - 3$.

Identify all critical points. where $f'(x) = 0$ or d.n.e.

$y' = 6x^2 + 6x - 12 = 6(x^2 + x - 2) = 6(x+2)(x-1) = 0 \Rightarrow \underline{x = 1, x = -2}$ are critical points

State the intervals over which the graph is increasing, decreasing.

		-2		1	
f'	$+$		$-$		$+$
f	$\nearrow$		$\searrow$		$\nearrow$

increasing $(-\infty, -2)$ and $(1, \infty)$

decreasing $(-2, 1)$

Identify any ~~absolute or~~ relative extrema.

$x = -2$ is rel. max

$x = 1$ is rel. min

Do the same for $y = x^3 - 9x^2 + 15x - 4$.

$y' = 3x^2 - 18x + 15$

Identify all critical points.

$f'(x) = 3x^2 - 18x + 15 = 3(x^2 - 6x + 5) = 3(x-1)(x-5) = 0$

$\Rightarrow x = 1, x = 5$ are critical!

State the intervals over which the graph is increasing, decreasing.

		1	5
f'	+	−	+
f	↗	↘	↗

increasing: $(-\infty, 1) \cup (5, \infty)$

decreasing $(1, 5)$

Identify any absolute or relative extrema.

$x=1$ is max, $x=5$ is min

A supermarket manager wants to establish an inventory policy for frozen orange juice. He finds that his inventory costs each month are

$$C(x) = \frac{360000}{x} + 4x$$ dollars, where x is the number of cases of orange juice.

How many cases should he order each month to minimize his inventory costs?

Produce 300 cases, since

$C'(x) = -360000x^{-2} + 4 = 0$

$4x^2 = 360000$

$x^2 = 90000$

$\Rightarrow x = \pm 300$

		−300	300
C'	+	−	+
C		↘	↗

$\Rightarrow x = 300$ is a rel. minimum

The analysis of the daily output of a factory assembly line shows that about

$H(t) = 60t + t^2 - t^3$ units are produced after t hours of work.

The factory currently operates 4 hours a day but management is thinking about operating it a little longer. Would the output increase or decrease?

$H'(t) = 60 + 2t - 3t^2$

$\Rightarrow H'(4) = 60 + 8 - 32 > 0$

$\Rightarrow H'(t) > 0$ for $t = 4$

$\Rightarrow$ H is increasing when $t = 4$ $\Rightarrow$ operate longer than 4 hours and the production increases

The Consumer Price Index (CPI) of an economy is described by the function

$I(t) = 200 + 3t - 0.4t^2$, where t is time in years and t = 0 corresponds to the year 2004.

Will the CPI increase in 2010?

$I'(t) = 3 - 0.8t$. At 2010, $t = 6$ $\Rightarrow I'(6) = 3 - 4.8 < 0$

$\Rightarrow I(t)$ will decrease!

Suppose the cost function of making q throw rugs is

$$C = 4q^2 - 2\sqrt{q^3} + 4400.$$

Find the marginal cost function

$C'(q) = \underline{8q - \frac{2}{3} q^{-2/3}}$ is marginal cost

as well as the marginal cost for q = 3.

$C'(q=3) = 8 \cdot 3 - \frac{2}{3} \cdot \frac{1}{9} = \underline{24 - \frac{2}{27}}$

What does that mean?

The cost for producing one more rugs than 3 will increase by about $\underline{\underline{23.92}}$

Find the fixed cost.

$C(0) = 4400$ is fixed cost

What does *that* mean?

Suppose the cost for producing q items is

$$C(q) = 6q^3 - 320q + 1700.$$

Find the marginal cost function

$C'(q) = 18q^2 - 320$

as well as the marginal cost for q = 2.

$C'(2) = 18 \cdot 4 - 320$
$= -248 \Rightarrow$ Cost for increased production will go down!

What does that mean?

Cost for increased production level will go down!

Find the fixed cost.

$C(0) = 1200$

What does *that* mean?

Means the cost for producing no units is 1200.