

### MATH 1203 – Practice Exam 3

*This is a practice exam only. The actual exam may differ from this practice exam.*

---

Please provide brief answers to the following questions:

- a) If you are using a t-distribution with  $df = 11$  for a statistical test, and the number  $t_0$  you compute is  $t_0 = 2.10$ . What is your conclusion for the corresponding test?

**$p = 2 * P(t \geq 2.1) = 2 * 0.029 = 0.058$  is greater than 0.05 so inconclusive**

- b) If you are using z-distribution for a statistical test at the usual 0.05 (= 5%) level of significance, the number  $z_0$  you compute is  $z_0 = 2.89$ , and the corresponding p-value for that value of  $z_0$  is 0.038. What is your conclusion for the corresponding test?

**Reject null hyp.**

- c) Someone is interested in designing a statistical test for the mean of a population. In deciding whether to use a test based on the t-distribution or a test based on the standard normal distribution, what is the deciding factor?

**Sample size**

- d) If you are using a t-distribution with  $df = 24$  for a statistical test at the  $\alpha = 0.05$  level (which means that you will reject the null hypothesis if the p-value is less than  $\alpha = 0.05$ ), and the value of  $t_0 = 2.15$ , what is your conclusion?

**(using  $df = 24$ ):  $p = 2 * P(t \geq 2.15) = 2 * 0.02 = 0.04$  is less than 0.05 so reject null hyp.**

- e) If you are using a t-distribution with  $df = 5$  for a statistical test, and the number  $t_0$  you compute is  $t_0 = 2.15$ , what is your conclusion for the corresponding test now? Compare that with the previous question and explain the difference in your conclusions.

**(using  $df = 5$ ):  $p = 2 * P(t \geq 2.15) = 2 * 0.04 = 0.08$  is bigger than 0.05 so inconclusive. That makes sense, since in the prev. example the sample mean is based on 25 data points, so there is less opportunity that it's based on chance than in this example, where the sample size would be 6. So in the prev. case the difference of 2.15 is more significant (so I comfortably reject null hyp.), but in this case the same difference could be due to chance, so I decide on inconclusive.**

- f) If you are using z-distribution for a statistical test at the usual 5% level of significance, the number  $z_0$  you compute is  $z_0 = 1.64$ , and the corresponding p-value for that value of  $z_0$  is  $p = 0.0505$ . What is your conclusion for the corresponding test?

**Inconclusive**

- g) You are conducting a statistical test for the population mean at the  $\alpha = 0.05$  level. The null hypothesis is  $H_0 = 17.1$ , while the alternative hypothesis is  $H_a \neq 17.1$ . The sample size is large enough to use a normal distribution, and the statistics for the sample turns out to be  $z_0 = 2.045$ . From the standard normal table for the z-distribution you compute  $P(z > 2.045) = 0.0202$ . What is the p-value, and what is your conclusion based on that value?

**$p = 2 * P(z_0 > 2.045) = 2 * 0.02 = 0.04$  is less than 0.05, so you reject the null hypothesis  $H_0$**

- h) A statistical test for the population mean at the  $\alpha = 0.05$  level results in your rejection of the null hypothesis. Can the null hypothesis still be true? If so, what is the probability that the null hypothesis is true, even though you rejected it?

**Yes it can, with probability less than  $\alpha = 0.05$ . If we computed a p-value, then that number would be the probability that  $H_0$  could be true even though I would reject it.**

- i) In a statistical test, is there ever a situation where you *accept* the null hypothesis?

**nope**

---

2. You are conducting a statistical test for the population mean at the  $\alpha = 0.05$  level. The null hypothesis is  $H_0 = 81.5$ , while the alternative hypothesis is  $H_a \neq 81.5$ . The sample size is  $n = 22$  and the statistics for the sample turns out to be  $t_0 = 1.6045$ . What is your conclusion?

$$p = 2 * P(t > 1.6045) = 2 * 0.06 > 0.05 \text{ so inconclusive}$$

---

You were asked to compute a 95% confidence interval. The resulting interval, however, turned out to be too large to be of use to your client. What could you do to achieve a smaller confidence interval?

**You could compute a 90% confidence interval, or repeat the experiment with an increased sample size.**

---

On average, do males outperform females in mathematics? To answer this question, psychologists at the University of Minnesota compared the scores of male and female eighth-grade students who took a basic skill math test. A summary of the test scores is displayed below.

	<b>Males</b>	<b>Females</b>
<b>Sample Size</b>	1764	1739
<b>Mean</b>	48.9	48.4
<b>Standard Deviation</b>	12.96	11.85

$$H_0: \text{mean}_1 = \text{mean}_2$$

$$H_1: \text{mean}_1 \text{ not equal to } \text{mean}_2$$

$$z_0 = (48.9 - 48.4) / s, \text{ where } s = 0.41948220487302895, \text{ so that}$$

$$z_0 = 1.192$$

Therefore  $p = 2 * P(z > 1.192) = 2 * 0.1170$  is not less than 0.05, so Test is Inconclusive

---

5. The Cleveland Casting plant produces iron automotive castings for Ford. When the process is stable, the target pouring temperature of the molten iron is 2,550 degrees. The pouring temperatures for a random sample of 10 crankshafts produced at the plant are listed below. Does the mean pouring temperature differ from the target setting?

2543, 2541, 2544, 2620, 2560, 2559, 2562, 2553, 2552, 2553

We can compute the sample mean  $\bar{x} = 2558.7$  and the sample standard deviation  $s = 22.7$ . Thus:

$$H_0: \text{mean} = 2550$$

$$H_1: \text{mean not equal to } 2550$$

$$t_0 = (2558.7 - 2550) / (22.7 / \sqrt{10}) = 1.212$$

$p = 2 * P(t > 1.212)$  will be less than 0.05 so our test is inconclusive, i.e. there is not enough evidence to suggest anything wrong.

---

6. According to USA Today (Dec. 1999) the average age of MSNBC TV News viewers is 50 years. A company wants to market a product for this age group, but wants to ensure that the USA Today study is correct before investing advertisement money. They select 50 US households at random that view MSNBC TV News and find their average age to be 53.8 years with a standard deviation of 7.1 years. Should the company invest in advertising?

$$H_0: \text{mean} = 50$$

$$H_a: \text{mean not equal to } 50$$

$$z_0 = (53.8 - 50) / (7.1 / \sqrt{50}) = 3.78$$

$p$  is certainly less than 0.05, so this test results in rejecting the null hyp. Thus, do not advertise!

---

7. The "fear of negative evaluation" (FNE) scores for 11 bulimic female students and 14 normal female students are shown below (the higher the score, the greater the fear of negative evaluation). What is the average FNE score of bulimic female students and that of normal female students? Is there a significant difference between the mean FNE scores?

**Bulimic students:** 21, 13, 10, 20, 25, 19, 16, 21, 24, 13, 14

**Normal students:** 13, 6, 16, 13, 8, 19, 23, 18, 11, 19, 7, 10, 15, 20

We compute  $\bar{x}_1 = 17.8$ ,  $s_1 = 4.9$  and  $\bar{x}_2 = 14.1$ ,  $s_2 = 5.3$

**H<sub>0</sub>: mean<sub>1</sub> = mean<sub>2</sub>**

**H<sub>a</sub>: mean<sub>1</sub> not equal to mean<sub>2</sub>**

**t<sub>0</sub> = (17.8 – 14.1) / s where s = 2.047 so that**

**t<sub>0</sub> = 1.807**

**p = 2\*P(t > 1.807) will be greater than 0.05. Thus, the test is inconclusive.**

---

8. Suppose you want to compare a new method of teaching reading to “slow learners” to the current standard method. You select a random sample of 22 slow learners; 10 of them are taught by the new method and 12 are taught by the standard method, for the same period of time. The reading scores for the two groups were as follows:

New Method	Standard Method
80, 80, 79, 81, 76, 66, 71, 76, 70, 85	79, 62, 70, 68, 73, 76, 86, 73, 72, 68, 75, 66

- What is the difference in average reading scores between the two methods?
- Conduct a test to determine whether the new method is better than the standard method.

**We compute  $\bar{x}_1 = 76.4$  and  $s_1 = 5.8$  as well as  $\bar{x}_2 = 72.3$  and  $s_2 = 6.65$ . Because we will need it soon, let's also compute the combined  $s = \sqrt{(s_1^2/n_1 + s_2^2/n_2)} = 2.65$**

**Question a is  $76.4 - 72.3 = 4.1$**

**Question b involves a test as usual:**

**H<sub>0</sub>: tests are the same (means are equal)**

**H<sub>1</sub>: tests are different (means not equal)**

**t<sub>0</sub> = 4.1 / 2.65 = 1.54**

**This will again result in an inconclusive test (too bad, sorry – hopefully we'll get a conclusive test soon)**

---

The lifetimes (in years) of ten automobile batteries of a particular brand are:

2.4	1.9	2.0	2.1	1.8
2.3	2.1	2.3	1.7	2.0

Estimate the mean lifetime for all batteries, using a 95% confidence interval.

**Mean = 2.06**

**S = 0.227**

**Std error = 0.0718**

**Multiplier: use df = 9 and compute P(t > \_\_\_ ) = 0.05 to get multiplier = 2.262157**

**So from  $2.06 \pm 0.0718 * 2.262157 = 2.06 \pm 0.1624$  which is**

**From 1.8975 to 2.2224**

**Or compute the confidence interval in one click, comes out the same.**

---

9. A large supermarket chain sells longhorn cheese in one-pound (= 16 ounces) packages. As a city inspector you weigh 81 randomly selected packages of cheese and note that the sample mean is 15.58 ounces, with a standard deviation of 1.44 ounces. You therefore suspect that the chain is miss-labeling the cheese and that the actual weight of a package is less than 16 ounces. Use this data to test your suspicion against the null hypothesis that the average weight of a package is 16 ounces. Use  $\alpha = 0.05$ .

**H<sub>0</sub>: mean = 16**

**H<sub>a</sub>: mean not equal to 16**

**z<sub>0</sub> = (15.58 – 16) / (1.44 / sqrt(81)) = -2.625**

**Now we compute  $p = 2*P(z > 2.625) = 2*0.0044 = 0.0088 < 0.05$  so we reject the null hypothesis. In other words, we think that the cheeses are mislabeled!**

---

10. A test was conducted to determine the length of time required for a student to read a specified amount of material. All students were instructed to read at the maximum speed at which they could still comprehend the material. Sixteen students took the test, with the following results (in minutes):

25, 18, 27, 29, 20, 19, 25, 24, 32, 21, 24, 19, 23, 28, 31, 22

Estimate the mean length of time required for all students to read the material, using a 95% confidence interval.

**We compute  $\bar{x} = 24.18$  and  $s = 4.32$ . The multiplier works out to 2.131. Thus the 95% confidence interval goes from  $24.18 - 2.131 * 4.32 / \sqrt{16} = 21.87$  to  $24.18 + 2.131 * 4.32 / \sqrt{16} = 26.48$**

---

11. A group of 26 rats was selected for a study to test a new drug. Each rat's heart rate was measured prior to receiving that drug and again 2 hours later. The sample mean drop in blood pressure between the readings was 28.2, and the standard deviation was 10.0. You know from previous experiments that the average drop of blood pressure for all other available drugs is 25.0 in a similar setup. Use this data to test your claim that your drug will deliver a better performance than the previously available drugs. Use  $\alpha = 0.05$ .

**H<sub>0</sub>: mean = 25**

**H<sub>1</sub>: mean not equal to 25**

$$t_0 = (28.2 - 25) / (10.0 / \sqrt{26}) = 1.61$$

**That  $t_0$  value is not large enough (I can tell by now without looking up the probability) so the test is inconclusive.**

---

12. The caffeine content of a random sample of 90 cups of black coffee dispensed by a new machine is measured. The mean and standard deviation for the sample are 110 mg and 6.1 mg, respectively.

- a) Compute a 90% confidence interval for the true population mean caffeine content per cup dispensed by the machine.

**Whatever the answer might be ☺**

- c) If you would compute a 99% confidence interval for the true population, would it be wider or narrower than the 90% confidence interval? (You do **not** actually have to compute this interval to answer the question).

**A 99% confidence interval is bigger, or wider, than a 95% one,**

- d) Another person selected a random sample of 900 instead of 90 cups, and the mean and standard deviation of this larger sample turned out to be 110mg and 6.1mg as well. That person uses her data to compute a 90% confidence interval. Would the 90% confidence interval for the larger sample size be wider or narrower than the 90% confidence interval for the smaller sample size?

**For larger sample size the confidence interval will be narrower, or smaller**

---

13. To test the research hypothesis that teacher expectation can improve student performance, two groups of 61 students were compared. Teachers of the experimental group were told that their students would show large IQ gains during the test semester, while teachers of the control group were told nothing. At the end of the semester, IQ change scores were calculated with the following results:

	<b>Mean</b>	<b>Standard Deviation</b>	<b>Sample Size</b>
<i>Experimental</i>	16.5	14.2	61
<i>Control</i>	7.0	13.1	61

Test the null hypothesis of no effect on mean IQ change scores against the above research hypothesis.

**This looks like there is a significant difference between the means, doesn't it? Let's compute:**

**H<sub>0</sub>: means are the same**

**H<sub>a</sub>: means are not the same**

**The combined  $s = 2.47$**

$$z_0 = 16.5 - 7.0 / 2.47 = 3.8$$

**p will be less than 0.05, so we reject the null hypothesis, i.e. We think that teacher expectations can indeed improve student performance.**

---

14. Over the past five years the mean time for a warehouse to fill a buyer's order has been 26 minutes. Officials of the company believe that the length of time has increased recently, either due to a change in the work force or due to a change in customer purchasing policies. The processing time (in minutes) was recorded for a random sample of 121 orders processed over the past month. The mean of that sample is  $\bar{x} = 28.20$  and the sample standard deviation is  $s = 11.44$ . Does this data present sufficient evidence to indicate that the mean time to fill an order has increased, if your error is supposed to be no larger than 0.05? Justify your argument by setting up all four components of a statistical test.

**H<sub>0</sub>: mean = 26**

**H<sub>a</sub>: mean not equals to 26**

**$z_0 = (28.2 - 26) / (s / \sqrt{121}) = 2.122$**

**This will be close, so we compute  $p = 2 * P(z > 2.122) = 2 * 0.0174 = 0.0348$ , which is less than 0.5, so we reject the null Hypothesis (and accept the alternative). Thus there is sufficient evidence to indicate that the mean order has increased indeed.**

15. A poll of 100 US congress people was taken to determine their opinions concerning a bill to raise the ceiling on the national debt. Each congressperson was then classified according to political party affiliation and opinion on the policy. The results are summarized below. Since the vote is close one would like to find out whether congress voted along party lines or not. Therefore, please test the null hypothesis that these two classifications are independent of one another, against the alternative hypothesis that they are not. Use a level of significance of 0.05

	<i>approve of bill</i>	<i>do not approve of bill</i>	<i>non opinion yet</i>	<i>Total</i>
<i>Republican</i>	28 (22.09)	14 (19.74)	5 (5.17)	47
<i>Democrat</i>	19 (24.91)	28 (22.26)	6 (5.83)	53
<i>Total</i>	47	42	11	100

StatCrunch computes the Chi-Square distribution as follows:

#### Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	6.143 <sup>a</sup>	2	.046
Likelihood Ratio	6.222	2	.045
N of Valid Cases	100		

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 5.17.

**This is a Chi-Square test of independence.**

**H<sub>0</sub>: the two variables are not related (are independent)**

**H<sub>1</sub>: the two variables are related**

**chi\_square = 6.143 (according to StatCrunch)**

**p = 0.046, which is smaller than 0.05 so we reject H<sub>0</sub>, which means that the variables are related.**