Panel 1

Last time (A long time ago): we computed

$P(z > 0.75)$ for $z = N(0,1)$ ✓

$P(6 \le x \le 12)$ = for $x = N(8,4)$ ✓

$P(x \le 12) - P(x \le 6)$

$0.8413 - 0.3085 = \underline{0.54}$

Conversion from $N(\mu, \sigma)$ to z-score

$z = \frac{x-\mu}{\sigma}$

If $x = 2$, $x = N(3,5)$

→ z-value of 2 is $z = \frac{x-\mu}{\sigma} = \frac{2-3}{5} = \frac{-1}{5} = \underline{-0.2}$

Panel 2

Reverse lookup

$P(z \le z_0) = 0.25$

$N(0,1)$ ⟶ $z_0 = \underline{-0.674}$

$P(x \le x_0) = 0.7$ where $x = N(75,10)$

⟶ $\underline{81.245}$

Conversion from z to $N(x, \mu)$:

$z = \frac{x-\mu}{\sigma}$ or $z\sigma = x - \mu$

$x = \sigma \cdot z + \mu$

Panel 3

Central Limit Theorem – 'everything is normal'

Say we have a distribution of unknown shape, with mean μ and std. dev. σ.

Suppose we keep selecting samples of size n and compute the sample mean $\bar{x}$ each time

Then: The $\bar{x}$ have normal distribution with mean μ and std. dev. $\sigma/\sqrt{n}$

Short Form: $\bar{x}$ is $N(\mu, \frac{\sigma}{\sqrt{n}})$

Panel 4

What is avg. mpg for US cars?

MPG's have some distr. with some μ and σ

Survey cars, say $N = 400$, and check their mpg. Say I get sample mean $\bar{x} = 23.5$ and $s = 3.92$.

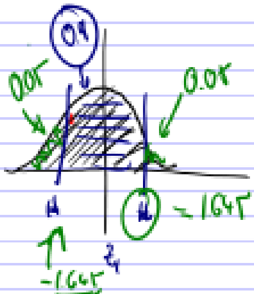
Again: 400 other cars, find $\bar{x} = 22.9$ (≈ 23.5)

Again: 400 cars find $\bar{x} = 24.1$ (≈ 23.5)

Know: $\bar{x}$ are $N(\mu, \sigma/\sqrt{n})$

Ex: Want to know $P(\# < \bar{x} < \#) = 0.9$

Panel 5



$P(z > \#) = 0.05$

'reverse lookup' finds

But these numbers are z-scores! $z = 1.645$. But $\bar{x}$ comes $N(\mu, \sigma/\sqrt{N})$ by CLT

$-1.645 = \frac{\bar{x}-\mu}{\sigma/\sqrt{N}} \Rightarrow \bar{x} = \mu \pm 1.645 \frac{\sigma}{\sqrt{N}}$

This is useful

Panel 6

If $+1.645$ $\quad x_1 = \mu + 1.645 \frac{\sigma}{\sqrt{N}}$

$-1.645 \Rightarrow x_2 = \mu - 1.645 \frac{\sigma}{\sqrt{N}}$

Back to example.

unknown mpg of all cars!

$N = 400$, $\bar{x} = 23.5$, $s = 2.82$. Find μ

$\mu - 1.645 \cdot \frac{\sigma}{\sqrt{N}} = \bar{x} - 1.645 \frac{s}{\sqrt{N}} =$

$= 23.5 - 1.645 \cdot \frac{2.82}{\sqrt{400}} = 22.95$

$\mu + 1.645 \cdot \frac{\sigma}{\sqrt{N}} = 23.5 + 1.645 \cdot \frac{2.82}{\sqrt{400}} = 24.14$

Conclusion: Know that avg mpg of all cars is between 22.95 and 24.14, with 90% certainty

Panel 7

Confidence Intervals

To find a 90% confidence interval about population mean μ

① Find $s/\sqrt{N}$ (standard error)

② Compute: $1.645 \cdot \frac{s}{\sqrt{N}}$

③ Answer: between $\bar{x} - 1.645 \cdot s/\sqrt{N}$ and

$\bar{x} + 1.645 \cdot s/\sqrt{N}$.

90% certain!

Panel 8

Ex: Coffee machine gives out coffee. Want to find avg. caffeine content.

Sample 81 cups, find $\bar{x} = 110$ mg with $\sigma = 5$ mg.

① Standard error: $s/\sqrt{N} = 5/\sqrt{81} = 5/9 = 0.555$

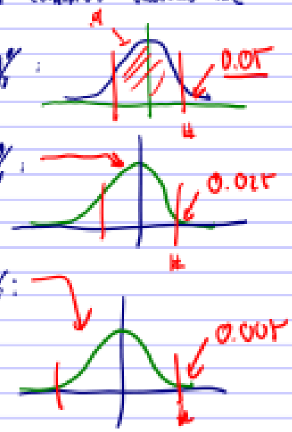
② Multiplier: $1.645 \cdot s/\sqrt{N} = 1.645 \cdot 0.555 = 0.9139$

③ Answer: μ is between 110 ± 0.9139 mg, or from 109.0861 to 110.9139

with 90% certainty

Panel 9

Other common levels are



90%: $\Rightarrow A = 1.645$

95%: $\Rightarrow A = 1.96$

99%: $\Rightarrow A = 2.58$

Panel 10

Confidence Interval about μ

① Find $\frac{\sigma}{\sqrt{N}} \cdot \frac{s}{\sqrt{N}}$

② Multiplier

90%. $1.645 \frac{s}{\sqrt{N}}$

95%. $1.96 \cdot s/\sqrt{N}$

99%. $2.58 \cdot s/\sqrt{N}$

③ Answer.

$\bar{x} \pm (\quad)$

I am 90% or 95% or 99% certain!

Panel 11

Ex: The active ingredients of some medication is measured in ppm. A random sample gives.

10, 11, 9.5, 7, 10.9, 11.5, 12.7, 10, 9.8

Find an estimate for the unknown population mean μ.

The usual confidence interval is 95%

$\bar{x} = 10.267$, $s = 1.581$

$\Rightarrow$ std error $= \frac{1.581}{\sqrt{9}} = \frac{1.581}{3} = 0.527$

the width: $1.96 \cdot 0.527 = 1.033$

10.267 − 1.33

μ is 10.267 ± 1.033, or μ is between 9.237 and 11.51

Panel 12

Simpler: On Phone

1) enter #'s

2) Stats → Z-Stats → One Sample → with data

check ☐ Confidence interval

hit Compute

L. Limit: 9.23

U. Limit: 11.29

Panel 13

Ex: To fit soldiers with new helmets, we need to know their avg. head size.

Get 1000 soldiers, find, say, that $\bar{x} = 52$ cm, $s = 8.5$

Want to find μ with 99% certainty.

std error: $8.5/\sqrt{1000} = 0.2688$

multiplier: $2.57 \cdot 0.2688 = 0.6927$

μ equals 52 ± 0.6927 or

μ between 51.317 and 52.693, 99% sure of it

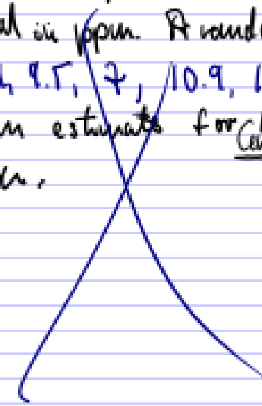
Panel 14

Ex: The active ingredients of some medication is measured in ppm. A random sample gives:

10, 11, 9.5, 7, 10.9, 11.5, 12.7, 10, 9.8

Find an estimate for the unknown population mean μ.

Central Limit Theorem: if we have a ... we select samples of size N they will have a



Panel 15

Is 90% or 99% bigger?

Want to find, or estimate, μ.

With 100% certainty, μ is between $-\infty, \infty$

99% sure: 5 to 11

95% is smaller than (5, 11), say (6, 10)

90% is smaller still $\approx (8, 9)$

1% (8.49, 8.51)

Q's: ① $P(z \le 1.6) = ?$ ③ confidence intervals

② $P(z \ge 7) = 0.5$