

Panel 1

Last Time

Computed probabilities for $N(0,1)$:

- $P(Z < 1) \stackrel{>0.5}{\circlearrowleft} \stackrel{0.1}{\circlearrowright} = 0.8413$
- $P(Z < -1) \stackrel{<0.5}{\circlearrowleft} \stackrel{0.2}{\circlearrowright} = 0.1586$
- $P(Z > 0.75) \stackrel{<0.5}{\circlearrowleft} \stackrel{0.3}{\circlearrowright} = 0.2266$

$P(a < Z < b) = P(Z < b) - P(Z < a)$

$a=0.2 \quad b=1.8 \quad = 0.9640 - 0.5793 = 0.3847$

Panel 2

Computed Probabilities for $N(\mu, \sigma)$:

$P(X \geq 15) \stackrel{<0.5}{\circlearrowleft} = 0.1586$

$X \sim N(25, 10)$

$P(X \leq -15)$

$X \sim N(-20, 5) = 0.9413$

$P(2.5 < X < 5.5)$

$X \sim N(4, 1) = 0.75$

Panel 3

Convert X-score, X in $N(\mu, \sigma)$, to Z-score, Z in $N(0, 1)$

$$Z = \frac{X - \mu}{\sigma}$$

If X in $N(8, 3)$, then

$$P(X > \underline{10}) = P(Z \geq \underline{0.6667})$$

X-score of 10 has a Z-score of $\frac{10 - 8}{3} = \frac{2}{3} = \underline{0.6667}$

$$P(X > \underline{10}) = 0.2525$$

||

$$P(Z > \underline{0.6667}) = 0.2525$$

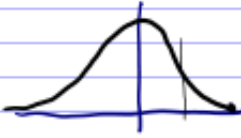
Q: Find Z-score for $x = 10$
if X in $N(8, 3)$: $Z = 0.6667$

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Panel 4

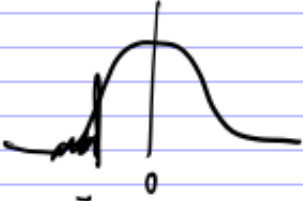
Reverse Probability $z_0 = \underline{1.0364}$

a) Find z_0 s.t. $P(Z \geq z_0) = 0.15$



b) z_0 s.t. $P(Z \leq z_0) = \underline{0.05}$

↳ $z_0 = \underline{-1.645}$

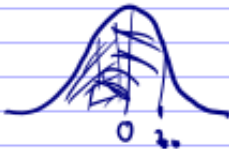


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Panel 5

Find z_0 s.t. $P(Z \leq z_0) = 0.75$. Then find the equivalent x_0 -value, $X = N(89, 12)$, i.e. find x_0 s.t. $P(X \leq x_0) = 0.75$, $X = N(89, 12)$

1.)



$z_0 = 0.6745$

Know: $z = \frac{x - \mu}{\sigma}$

$\sigma z = x - \mu$

$\mu + \sigma z = x = 89 + 12 \cdot 0.6745 = 97.094$

97.0938 by using normal calc

Panel 6

To convert x -score to z -score: $z = \frac{x - \mu}{\sigma}$

To convert z -score to x -score: $x = \mu + \sigma z$

Ex: Find prob. that a randomly selected adult in US is 57 years or older.

Know from GSS: AGE has $\bar{x} = 47.21$, $\sigma = 17.31$.

$\Rightarrow P(X > 57) = 0.2901$

$N(47.21, 17.31)$

This assumes that X is normal!!!

Panel 7

Prob. that a random person is 50 years or older.

$$P(X \geq 50), \quad X \text{ has } \mu = 47.71, \sigma = 17.35$$

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Panel 8

Central Limit Theorem. Say we have a distribution of unknown shape. If we select samples of size n and compute the sample mean \bar{X} , they will have a normal distribution (approx).

If original distribution has mean μ and std dev. σ , then the \bar{X} are Normal with mean μ and std dev. $\frac{\sigma}{\sqrt{n}}$ i.e.

$$\bar{X} \text{ are } N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

↳ Standard Error

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Panel 9

What is avg. mpg for US cars?

Survey cars, say $N=400$, and check their mpg. Say I get
sample mean $\bar{x} = 23.5$ and $s = 7.92$.

Do this again: $\bar{x} = 21.9$, $s = 7.99$

again $\bar{x} = 24.9$, $s = 8.01$

By CLT all samples are $N(\mu, \sigma/\sqrt{n})$

Ex: Want to know $P(\# < \bar{x} < \#) = 0.9$