Panel 1

Last Times

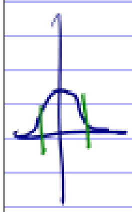
3x flips of 1 coin.

Finding probabilites.

$P(\text{at least } 2H)$ TTH

HHH, HHT, HTH, HTT, THH, THT, TTT

- flipping coins $\underset{2}{\_}\ \underset{2}{\_}\ \underset{2}{\_}$ $P(E) = 4/8 = 1/2$
- rolling dice: $P(\text{at least 4 in one die}) = 3/6 = 1/2$
- Standard normal distribution $N(0,1)$:



$$P(\underset{0.0630}{-1.53} < z < \underset{0.0150}{2.17}) = \underline{1 - 0.0630 - 0.0150}$$

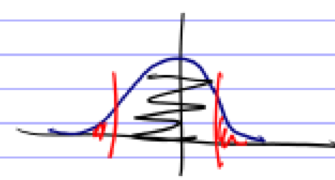
- Normal distribution $N(\mu, \sigma)$:

$P(90 < x < 100)$, x in $N(80, 20)$

Panel 2

$P(90 < x < 100)$, x in $N(80, 20)$

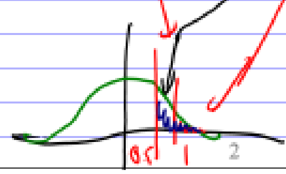
(1) Convert to z-scores.



$$z = \frac{90 - 80}{20} = 0.5$$

$$z = \frac{100 - 80}{20} = 1.0$$

$$P(90 < x < 100) = P(\underset{0.3085}{0.5} < z < \underset{.1587}{1.0}) = \underline{0.3085 - 0.1587}$$



Panel 3

Reverse Lookup

z is $N(0,1)$. Find z_0 such that

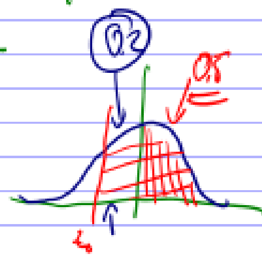
a) $P(z \geq z_0) = 0.3$ ($z \approx 0.525$) (Prev.: $P(z \geq 0.7) = ?$)

edge of table

inside the table

b) $P(z \geq z_0) = 0.7$

$P(z \geq \underline{-0.84}) = \underline{0.7}$



c) $P(X \geq X_0) = 0.2$ if X is $N(26, 9)$

First: $P(z \geq z_0) = 0.2 \Rightarrow \underline{z_0 = 0.84}$

Also: $z = \frac{x-26}{9} \Rightarrow 0.84 = \frac{x-26}{9} \Rightarrow 0.84 \cdot 9 + 26 = X$

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Panel 4

Central Limit Theorem: Say we have a distribution of unknown shape. If we select samples of size N and compute the sample mean $\bar{x}$, they will have a normal distribution

If original distribution has mean μ and std dev. σ, then the $\bar{x}$ are normal with mean μ and std. dev. $\sigma/\sqrt{N}$, ie

$$\bar{X} \text{ are } N\left(\mu, \frac{\sigma}{\sqrt{N}}\right)$$

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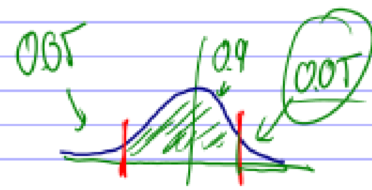
Panel 5

What is avg. mpg for US cars? Looking for μ!

Survey cars, say $N = 400$, and check their mpg. Say I get sample mean $\bar{x} = 23.5$ and $s = 7.82$.

Pick 400 other cars, find $\bar{x} = 23.8$, $s = 7.85$, close to before



Ex: Want to know $P(\# < \mu < \#) = 0.9$

-1.645 $\quad$ 1.645

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Panel 6

Found $P(-1.645 < z < 1.645) = 0.9$

$\bar{x}$ is $N\left(\mu, \frac{\sigma}{\sqrt{N}}\right)$

$$z = \frac{\bar{x} - \mu}{(\sigma/\sqrt{N})} \Leftrightarrow 1.645 = \frac{\bar{x} - \mu}{(\sigma/\sqrt{N})} \Rightarrow$$

$$\bar{x} = 1.645 \frac{\sigma}{\sqrt{N}} + \mu \Rightarrow \mu = \bar{x} - 1.645 \frac{\sigma}{\sqrt{N}}$$

also
$$\bar{x} = -1.645 \frac{\sigma}{\sqrt{N}} + \mu \Rightarrow \mu = \bar{x} + 1.645 \frac{\sigma}{\sqrt{N}}$$

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Panel 7

Confidence Intervals

To find a 90% confidence interval about population mean μ.

① Find $S/\sqrt{N}$ called Standard Error

② Compute: $1.645 \cdot \frac{S}{\sqrt{N}}$

③ Answer: μ is between

$$\bar{x} - 1.645 \cdot \frac{S}{\sqrt{N}} \quad \text{and} \quad \bar{x} + 1.645 \cdot \frac{S}{\sqrt{N}}$$

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Panel 8

Ex: Cars MPG. $N = 400$, $\bar{x} = 23.5$, $S = 7.82$

Want 90% confidence interval for unknown μ.

① $\frac{S}{\sqrt{N}} = \frac{7.82}{\sqrt{400}} = 0.391$

② $1.645 \cdot 0.391 = 0.643$

③ μ is between $\bar{x} - 1.645 \frac{S}{\sqrt{N}} = 23.5 - 0.643 = 22.856$

and $23.5 + 0.643 = 24.143$

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Panel 9

Ex: Coffee machine gives out coffee. Want to find avg. caffeine content.

Sample 81 cups, find $\bar{X} = 110$ mg with $\sigma = 5$ mg

① Standard error: $5/\sqrt{81} = 5/9 = \underline{0.555}$

② Multiplier: $1.645 \cdot 0.555 = \underline{0.914}$

③ Answer: μ is between $110 - 0.914 = \underline{109.086}$ and

with $\underline{90\%}$ certainty. $110 + 0.914 = \underline{110.914}$

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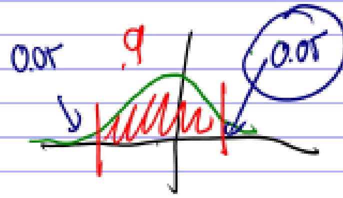
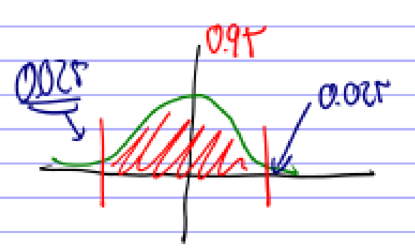
Panel 10

Other common intervals are

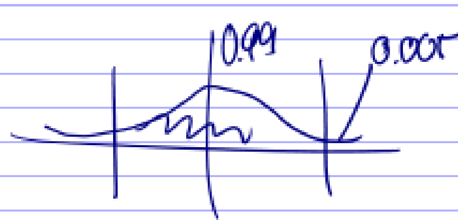
90%: $\bar{X} \pm 1.645 \frac{S}{\sqrt{N}}$



95%: $\bar{X} \pm \underbrace{1.96} \frac{S}{\sqrt{N}}$



99%: $\bar{X} \pm \underline{2.54} \frac{S}{\sqrt{N}}$



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Panel 11

Confidence Interval about μ

① Find std. error $S/\sqrt{n}$

② Multiplier:

90%	1.645
95%	1.96
99%	2.54

③ Answer. $\bar{x} \pm$ multiplier · std. error

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Panel 12

Large class of 170 000 students.
Select $n = 100$ exams, $\bar{x} = 75.9$, $S = 10.5$

90% conf. interval: $\bar{x} \pm 1.645 \cdot 1.05 <$ 77.6 / 74.2
between 74.2 and 77.6

99% conf. interval HW bigger interval than 90%

① std. error $\frac{S}{\sqrt{n}} = \frac{10.5}{10} = 1.05$

② Mult. is 1.645 (90%) or 2.54 (99%)

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