Panel 1

Last Time: Fourier Series

f is any periodic function with period P. Then

$$f(t) = a_0 + \sum_{j=1}^{\infty} a_j \cos\left(\frac{2\pi j t}{P}\right) + b_j \sin\left(\frac{2\pi j t}{P}\right)$$

Trick: find these values a_j and b_j

Problem: $a_j = \int f(t) \cos\left(\frac{2\pi j t}{P}\right) dt$ slow!

Trick: use some very smart algorithm called discrete Fast Fourier Transform (dFFT)

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Panel 2

History of FFT and Fourier Series

French Revolution 1789

Fourier

Born the son of an Auxerre tailor in 1768, Jean Baptiste Joseph Fourier lived to become one of France's greatest administrators, historians, and mathematicians. He graduated with honors from the military school in Auxerre, joined the staff of the Ecole Normale and then the Polytechnique in Paris. He went to Egypt with Napoleon, who made him Governor of Lower Egypt after the 1798 Expedition. After returning to France he was made Prefect at Grenoble in 1802 and then Baron in 1809, finally ending up as secretary of the Academie des Sciences in 1816 and Fellow in 1817. He contributed heavily to Description de l'Egypte, which covered the cultural and scientific results of Napoleon's invasion of Egypt, issuing 21 volumes over the period from 1808 to 1825. This work led to establishing Egyptology as a recognized new and separate discipline.

Today many areas of study benefit from Fourier analysis, however the method did not gain acceptability in Fourier's time. The reason behind this was the distrust of the use of series. One influential mathematician in 1828 commented: "Divergent series are the invention of the Devil, and it is shameful to base on them any demonstration whatsoever"[13]. Since then work by Dirichlet, Friedrich, Riemann, and others have resolved any doubts about the validity of the Fourier series.

The major setback in using the transform is computational time. The discrete Fourier transform (DFT) requires extensive computational time to evaluate. Even with the advent of the digital computer the techniques to reduce computational time were generally unknown until 1965 when James W. Cooley and John W. Tukey published their mathematical algorithm which has become known as the fast Fourier transform (FFT) [13].

http://me.oregonstate.edu/classes/me452/winter95/ButlerKeithMurphy/insth.html#A1.22

2

Panel 3

```
public static void computeFFT(int sign, int n, double ar[], double ai[])
    {
        double scale = 2.0 / (double) n;
        int i, j;
        for (i = j = 0; i < n; ++i)
        {
            if (j >= i)
            {
                double tempr = ar[j] * scale;
                double tempi = ai[j] * scale;
                ar[j] = ar[i] * scale;
                ai[j] = ai[i] * scale;
                ar[i] = tempr;
                ai[i] = tempi;
            }
            int m = n / 2;
            while ((m >= 1) && (j >= m))
            {
                j -= m;
                m /= 2;
            }
            j += m;
        }
        int mmax, istep;
        for (mmax = 1, istep = 2 * mmax; mmax < n; mmax = istep, istep = 2 * mmax)
        {
            double delta = sign * Math.PI / (double) mmax;
            for (int m = 0; m < mmax; ++m)
            {
                double w = m * delta;
                double wr = Math.cos(w);
                double wi = Math.sin(w);
                for (i = m; i < n; i += istep)
                {
                    j = i + mmax;
                    double tr = wr * ar[j] - wi * ai[j];
                    double ti = wr * ai[j] + wi * ar[j];
                    ar[j] = ar[i] - tr;
                    ai[j] = ai[i] - ti;
                    ar[i] += tr;
                    ai[i] += ti;
                }
            }
            mmax = istep;
        }
    }
```

magic happens to compute the output in these arrays ar, ai.

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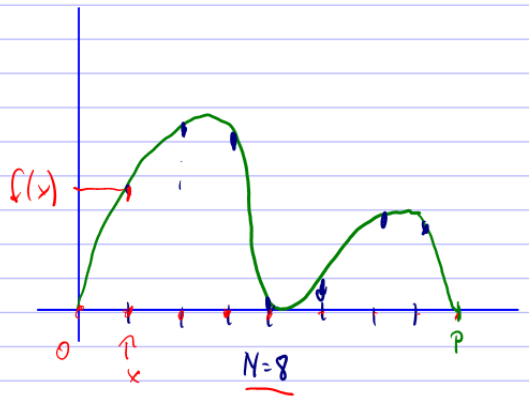
Panel 4

How to analyse a function:



① Divide interval [0, P] into N steps (sample rate)

② Evaluate function at those points

⇒ array ar

③ Create array ai of 0's

④ Call "computeFFT(1, N, ar, ai)

⑤ When done, ar, ai will contain answer:

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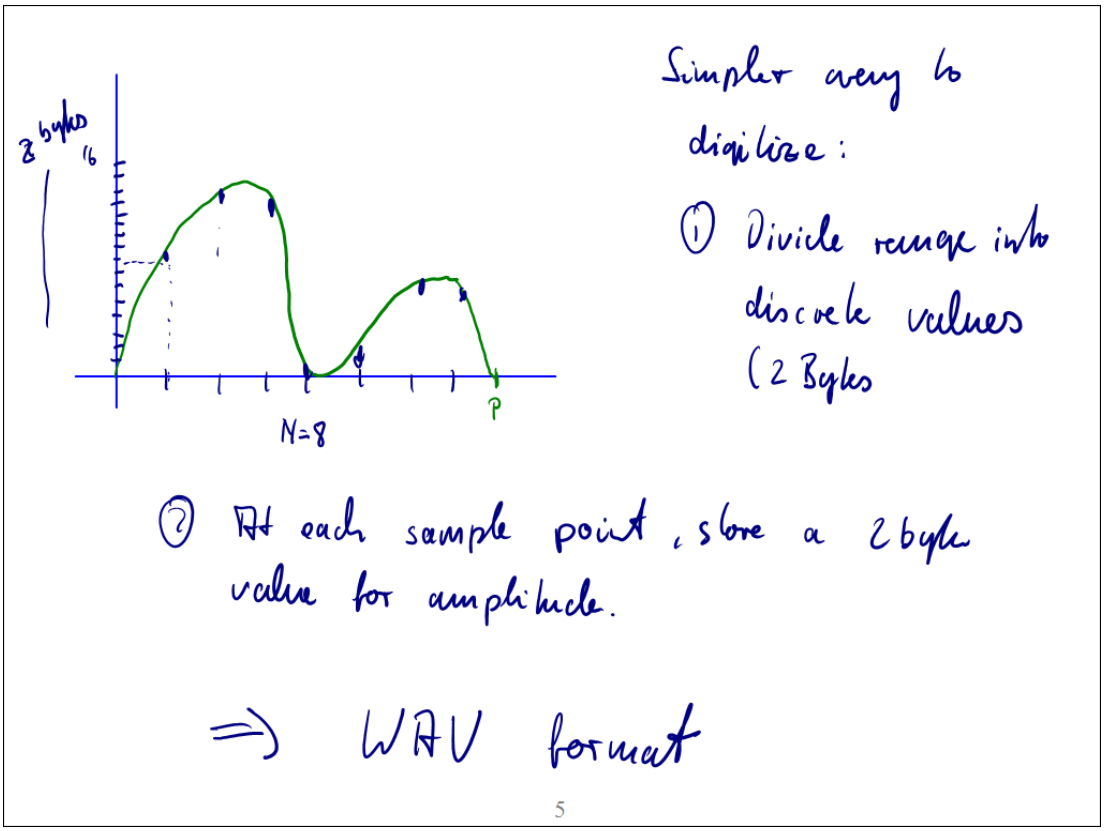
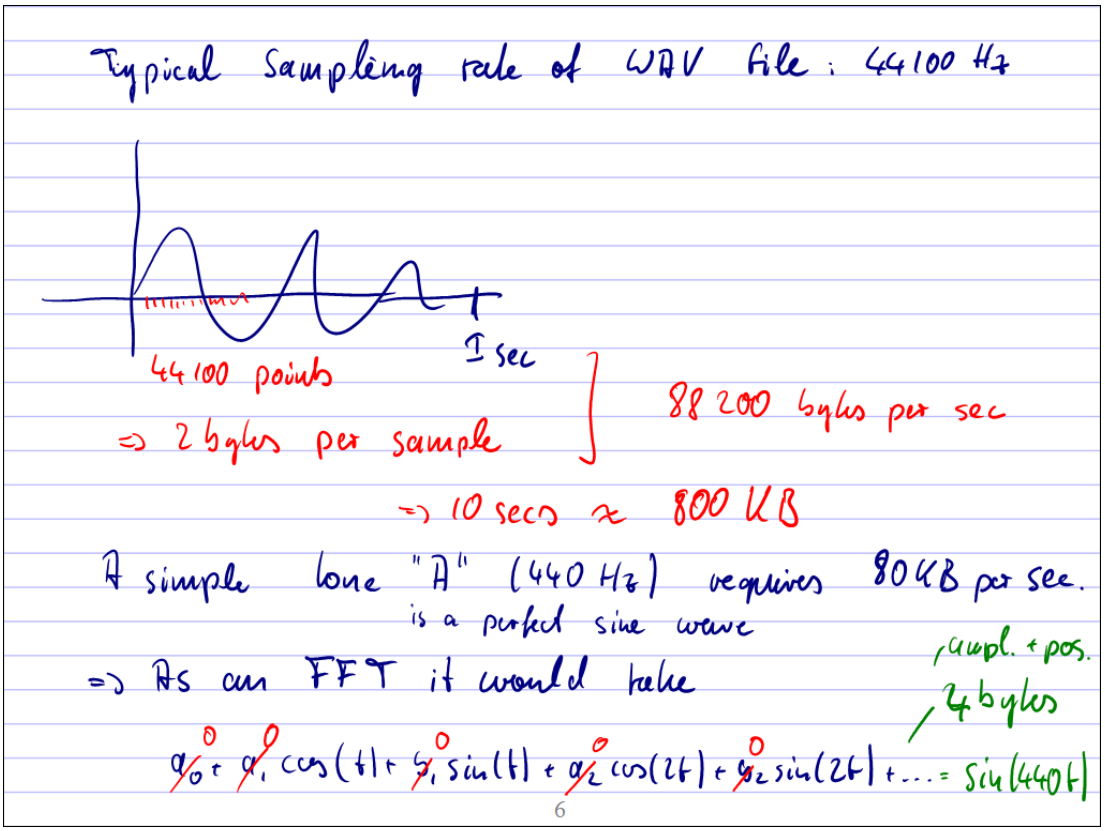
Panel 5



Panel 6



Panel 7

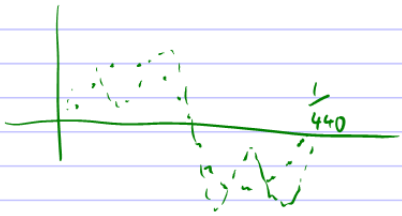
a) Goto Web page.

b) Download FFT.java

c) Start new Eclipse class!

Goal: I will give you an array representing a sound wave

=> you need to tell me which sounds



Specify later!

=> next Assignment!

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