

Panel 1

Lowest OSI Level is Physical Level
 \Rightarrow defines bits and handles
 their transmission.

Bits are digital signals: discrete range
 as opposed to analog signals: continuous range
 between 2 values

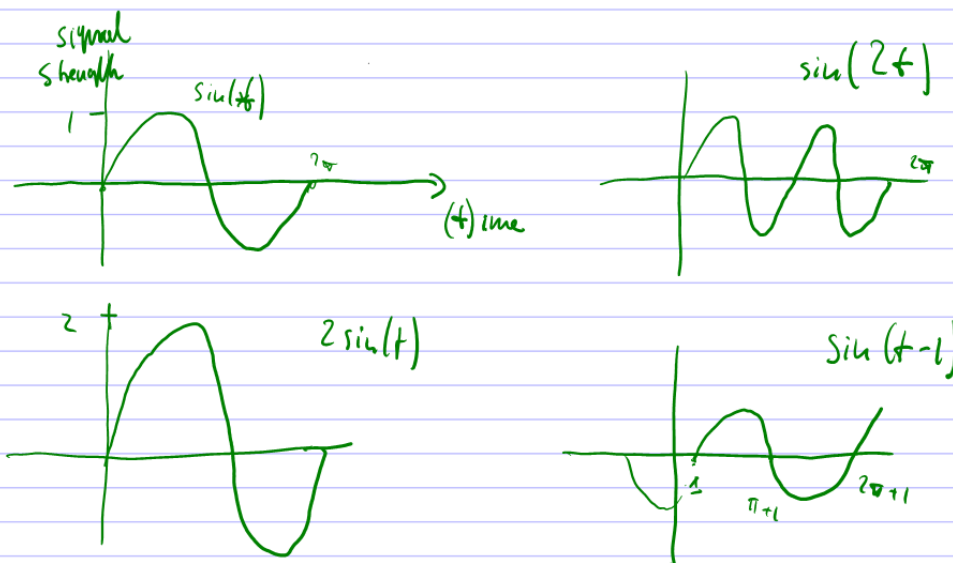
Start looking at analog signals

1

Panel 2

Analog Signals

Def. An analog signal is a continuously varying signal



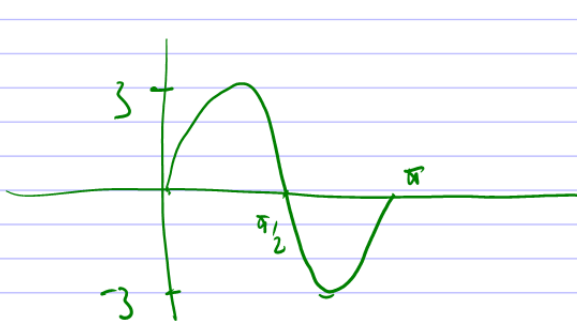
2

Panel 3

Analog Signals

Def: An ~~analog signal~~ ^{sin(t) has period} is a continuously varying signal

$$f = 1/\text{period} = (1/5 \text{ Hz})$$



$$3 \sin(\overbrace{2t}^{0 \dots 2\pi})$$

$$\text{frequency} = \frac{1}{\pi}$$

$$\text{Period: } \pi \text{ seconds}$$

$$\text{Amplitude: } 3$$

3

Panel 4

Period: time it takes for a ^{with period P} pattern to repeat

Frequency: # of times a signal oscillates per second

$$f(t) = a_0 + \sum_{j=1}^{\infty} \left(a_j \cos\left(\frac{2\pi j t}{P}\right) + b_j \sin\left(\frac{2\pi j t}{P}\right) \right)$$

Amplitude: largest and smallest value of pattern

$$a_j = \frac{1}{P} \int_{-P/2}^{P/2} f(t) \cos\left(\frac{2\pi j t}{P}\right) dt$$

$$b_j = \frac{1}{P} \int_{-P/2}^{P/2} f(t) \sin\left(\frac{2\pi j t}{P}\right) dt$$

4

Panel 5

~~Fourier theorem~~ $f(t) = x^2$
 Every periodic function can be expressed as
 is a sum of sin and cos
 a sum of sine and cosine functions:
 where

5

Panel 6

$f(t) = t^2$ is really a series sum of sin/cos
 $\Rightarrow t^2 = a_0 + a_1 \cos(t) + b_1 \sin(t) +$
 $+ a_2 \cos(2\pi t) + b_2 \sin(2\pi t) +$
 $+ a_3 \cos(4\pi t) + b_3 \sin(4\pi t) + \dots$
 $\Rightarrow t^2 = [a_0, a_1, b_1, a_2, b_2, a_3, b_3, \dots]$
 $t^2 = (1.5) + (3.7) \cos(t) - (1.9) \sin(t) +$
 $+ (8.5) \cos(2\pi t) + (5.2) \sin(2\pi t) +$
 $+ (8.9) \cos(4\pi t) + (0) \sin(4\pi t) + \dots$

6

Panel 7

Period: time it takes for a pattern to repeat

Frequency: # of times a signal oscillates per second

Amplitude: largest and smallest value of pattern

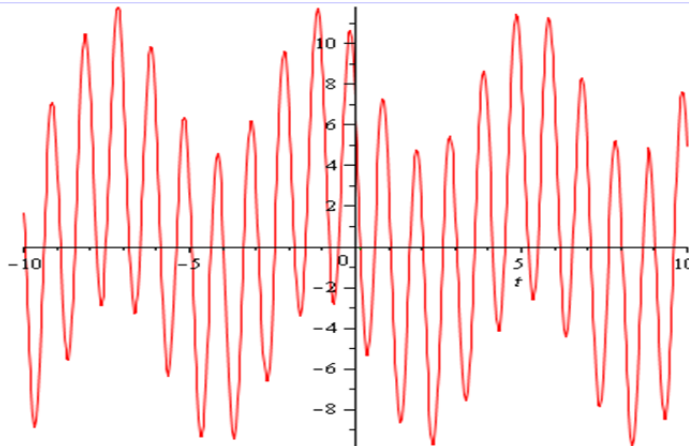
7

Panel 8

Ex. What is the function

$$\underline{[1, 2, -3, 4, -6, 0, 0, 0]} =$$

$$f(t) = 1 + 2 \cos(t) - 3 \sin(t) + 4 \cos(2\pi t) - 6 \sin(2\pi t)$$



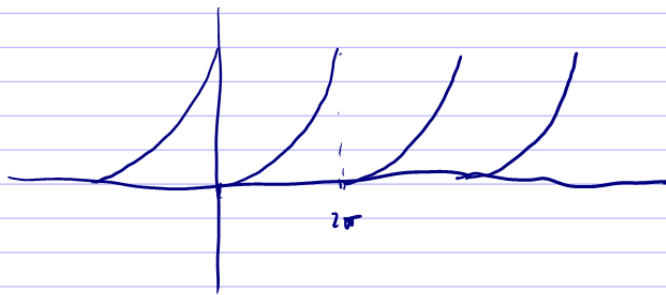
8

Panel 9

Other way: What is

$$f(t) = t^2 ? = [a_0, a_1, b_1, a_2, b_2, a_3, b_3, \dots]$$

$$a_0 + a_1 \cos(t) + b_1 \sin(t) + a_2 \cos(2\pi t) + b_2 \sin(2\pi t)$$



9

Panel 10

To compute these Fourier Coefficients, you could numerically integrate

$$a_j = \int f(t) \cos(jt) dt, \quad b_j = \int f(t) \sin(jt) dt$$

but that's too slow!

FFT algorithms to compute a_j, b_j

↳

quickly!

Fast Fourier Transform

10